

Quantum channel discrimination against jammers

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Abstract—We study the quantum channel discrimination between two channels with an adversary input party (a.k.a. a jammer), which interpolates between the best-case channel discrimination as studied by (Wang & Wilde, 2019) and the worst-case channel discrimination as studied by (Fang, Fawzi, & Fawzi, 2025), thereby generalizing both frameworks. To address this problem, we introduce the notion of minimax channel divergence and establish several of its key mathematical properties. We prove the Stein’s lemma in this new setting, showing that the optimal type-II error exponent in the asymptotic regime under parallel strategies is characterized by the regularized minimax channel divergence.

A full version of this paper is available at [1], which provides details and proofs of all results.

I. INTRODUCTION

Channel discrimination is a fundamental task in classical and quantum information theory [2]. The goal is to distinguish between two channels – conditional distributions in the classical case, or completely positive trace-preserving (CPTP) maps in the quantum case – by preparing suitable inputs and observing the outputs. Quantum channel discrimination underpins a wide range of applications, from foundational studies of quantum advantages [3]–[6] to quantum communication [7]–[12], quantum sensing [13], [14], and quantum biology [15], [16].

Most prior work focuses on the conventional *best-case* setting [3]–[6], [14], [17]–[24], where the tester has full control over inputs, intermediate operations, and measurements. Recently, a *worst-case* adversarial framework was introduced [25], motivated by quantum device verification [26], in which the adversary controls the input states while the tester only chooses measurements. These two extremes represent fundamentally different operational regimes.

In this work, we study *quantum channel discrimination against jammers*, a framework that interpolates between the best-case [27] and worst-case [25] scenarios. Our focus is on the asymmetric error exponent (Stein’s exponent) in the asymptotic regime. The study of channels with adversarial inputs dates back to classical models such as compound channels and arbitrarily varying channels (AVCs) [28]–[32]. Quantum generalizations include arbitrarily varying classical–quantum and quantum channels [33]–[37], as well as fully quantum arbitrarily varying channels [38], [39]. However, these works primarily address capacity problems rather than channel discrimination.

Technically, a channel with a jammer is modeled as a CPTP map $\mathcal{N}_{\text{AE} \rightarrow \text{B}} \in \text{CPTP}(\text{AE} : \text{B})$, where E denotes the jammer

system. Given two such channels $\mathcal{N}_{\text{AE} \rightarrow \text{B}}$ and $\mathcal{M}_{\text{AE} \rightarrow \text{B}}$, the tester prepares an input state ρ_{AR} and performs a measurement $0 \preceq M \preceq I_{\text{B}}$, while the jammer adversarially chooses a state on E. The resulting type-I and type-II error probabilities are defined via worst-case optimization over jammer inputs. For asymmetric hypothesis testing, we study the optimal type-II error (see Section IV and especially (42) for definitions)

$$\beta_{\epsilon}^{\text{E}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}), \quad (1)$$

and its asymptotic exponent under parallel channel uses,

$$\lim_{n \rightarrow \infty} -1/n \log \beta_{\epsilon}^{\text{E}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}^{\otimes n} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}^{\otimes n}). \quad (2)$$

Our main contribution is the introduction of the *minimax channel divergence*, together with its key properties. We show that the optimal type-II error is characterized by a hypothesis-testing version of this divergence, and we establish a Stein’s lemma for channel discrimination against quantum jammers, showing that the optimal error exponent is precisely given by the regularized minimax channel divergence. The result recovers known Stein’s lemmas in both best-case [27] and worst-case [25] limits.

The remainder of the paper is organized as follows. Section II introduces notation and preliminaries. Section III develops the minimax channel divergence and its properties. Section IV presents the result on the channel discrimination. Section V concludes the paper.

II. PRELIMINARIES

We introduce our notations and the definition of quantum divergence in this section.

- Given a quantum system A, its state space (which is a Hilbert space) is denoted by $\mathcal{H}_{\text{A}}$. We denote $\mathcal{L}(\mathcal{H}_{\text{A}})$, $\mathcal{H}(\mathcal{H}_{\text{A}})$, $\mathcal{H}_{+}(\mathcal{H}_{\text{A}})$, and $\mathcal{H}_{++}(\mathcal{H}_{\text{A}})$ the set of all linear operators, Hermitian operator, positive semidefinite operators, and positive definite operators on $\mathcal{H}_{\text{A}}$, respectively. We denote $\mathcal{D}(\mathcal{H}_{\text{A}})$ the set of density operators on $\mathcal{H}_{\text{A}}$.
- For two PSD operators on a same Hilbert space, i.e., $\rho, \sigma \in \mathcal{H}_{+}(\mathcal{H})$, $\rho \ll \sigma$ denotes the fact that the support of ρ is a subspace of the support of σ .
- We denote the set of all completely positive trace preserving (CPTP) maps from system A to system B by $\text{CPTP}(\text{A} : \text{B})$, and the set of all completely positive (CP) maps (not necessary trace preserving) from system A to system B by $\text{CP}(\text{A} : \text{B})$.

The remaining part of this section focuses on the concept of quantum divergence, i.e.,

Definition 1 (Quantum divergence). A functional $\mathbb{D} : \mathcal{D} \times \mathcal{H}_+ \rightarrow \mathbb{R}$ is called a quantum divergence if it satisfies the data-processing inequality, i.e., $\mathbb{D}(\mathcal{E}(\rho) \parallel \mathcal{E}(\sigma)) \leq \mathbb{D}(\rho \parallel \sigma)$ for any quantum channel $\mathcal{E}$, $\rho \in \mathcal{D}$, and $\sigma \in \mathcal{H}_+$. In particular, a quantum divergence satisfies the *direct-sum property* if

$$\mathbb{D}\left(\sum_x p_X(x) \cdot |x\rangle\langle x| \otimes \rho^{(x)} \parallel \sum_x p_X(x) \cdot |x\rangle\langle x| \otimes \sigma^{(x)}\right) = \sum_x p_X(x) \cdot \mathbb{D}\left(\rho^{(x)} \parallel \sigma^{(x)}\right). \quad (3)$$

for any random variable X and $\{\rho^{(x)}\}_x, \{\sigma^{(x)}\}_x \subset \mathcal{D}$.

It is noteworthy that the direct-sum property together with the data processing implies the joint convexity of such quantum divergence.

We define the quantum divergences between two sets of quantum states, which is at the center of discussion of this paper.

Definition 2. (Quantum divergence between two sets of states.) Given any quantum divergence $\mathbb{D}(\cdot \parallel \cdot)$, for any sets $\mathcal{A} \subseteq \mathcal{D}$ and $\mathcal{B} \subseteq \mathcal{H}_+$, the corresponding quantum divergence between these two sets of quantum states is defined as

$$\mathbb{D}(\mathcal{A} \parallel \mathcal{B}) := \inf_{\rho \in \mathcal{A}, \sigma \in \mathcal{B}} \mathbb{D}(\rho \parallel \sigma). \quad (4)$$

Note that if $\mathbb{D}$ is lower semi-continuous (which is true for most quantum divergences of interest), and $\mathcal{A}$ and $\mathcal{B}$ are compact sets, the infimum in the above expression is always attainable and can thus be replaced by a minimization [40, Theorem 7.3.1]. From a geometric perspective, this quantity characterizes the distance between two sets $\mathcal{A}$ and $\mathcal{B}$ under the “distance metric” $\mathbb{D}$. For any two sequences of sets $\mathcal{A} = \{\mathcal{A}_n\}_{n \in \mathbb{N}}$ and $\mathcal{B} = \{\mathcal{B}_n\}_{n \in \mathbb{N}}$, the *regularized* divergence is defined by

$$\mathbb{D}^\infty(\mathcal{A} \parallel \mathcal{B}) := \lim_{n \rightarrow \infty} \frac{1}{n} \mathbb{D}(\mathcal{A}_n \parallel \mathcal{B}_n), \quad (5)$$

provided the limit on the right-hand side exists.

III. QUANTUM MINIMAX CHANNEL DIVERGENCE

In this section, we develop the notion of *minimax channel divergence* for dual-input single-output quantum channels. Such notion is very useful in dealing with quantum channels with an adversary input.

Definition 3. Let $\mathbb{D}(\cdot \parallel \cdot)$ be a quantum divergence, and let A and B be two quantum systems. For any $\mathcal{N} \in \text{CPTP}(A : B)$ and $\mathcal{M} \in \text{CP}(A : B)$. The corresponding *best-case channel divergence* is defined as [41]

$$\mathbb{D}^\uparrow(\mathcal{N}_{A \rightarrow B} \parallel \mathcal{M}_{A \rightarrow B}) := \sup_{\rho_{AR} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_R)} \mathbb{D}(\mathcal{N}_{A \rightarrow B}(\rho_{AR}) \parallel \mathcal{M}_{A \rightarrow B}(\rho_{AR})), \quad (6)$$

where R is a reference system of arbitrary dimension. The *worst-case channel divergence* is defined as [25]

$$\mathbb{D}^\downarrow(\mathcal{N}_{A \rightarrow B} \parallel \mathcal{M}_{A \rightarrow B}) := \inf_{\sigma_A, \omega_A \in \mathcal{D}(\mathcal{H}_A)} \mathbb{D}(\mathcal{N}_{A \rightarrow B}(\rho_A) \parallel \mathcal{M}_{A \rightarrow B}(\omega_A)). \quad (7)$$

Definition 4. Let $\mathbb{D}(\cdot \parallel \cdot)$ be a quantum divergence, and let A , B and E be quantum systems. For any $\mathcal{N} \in \text{CPTP}(AE : B)$ and $\mathcal{M} \in \text{CP}(AE : B)$, the corresponding *minimax channel divergence* is defined as

$$\mathbb{D}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B} \parallel \mathcal{M}_{AE \rightarrow B}) := \sup_{\rho_{AR} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_R)} \inf_{\sigma_E, \omega_E \in \mathcal{D}(\mathcal{H}_E)} \mathbb{D}(\mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \parallel \mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \omega_E)), \quad (8)$$

where R is a reference system of arbitrary dimension. Furthermore, the induced regularized minimax channel divergence is defined as

$$\mathbb{D}^{\text{mx}, \infty}(\mathcal{N}_{AE \rightarrow B} \parallel \mathcal{M}_{AE \rightarrow B}) := \lim_{n \rightarrow \infty} \frac{1}{n} \mathbb{D}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B}^{\otimes n} \parallel \mathcal{M}_{AE \rightarrow B}^{\otimes n}), \quad (9)$$

provided that the limit exists.

Remark 5. The minimax channel divergence defined above interpolates between the best-case channel divergence (i.e., $\dim \mathcal{H}_E = 1$) and the worst-case channel divergence (i.e., $\dim \mathcal{H}_A = 1$). As an immediate observation, one can express the minimax channel divergence in terms of the worst-case channel divergence, i.e.,

$$\mathbb{D}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B} \parallel \mathcal{M}_{AE \rightarrow B}) = \sup_{\rho_{AR}} \mathbb{D}^\downarrow(\mathcal{N}_{E \rightarrow B}^{\rho_{AR}} \parallel \mathcal{M}_{E \rightarrow B}^{\rho_{AR}}) \quad (10)$$

where, for any $\mathcal{C}_{AE \rightarrow B} \in \text{CP}(AE : B)$, denote the induced channel $\mathcal{C}_{A \rightarrow B}^{\rho_A}(\sigma_E) := \mathcal{C}_{AE \rightarrow B}(\rho_A \otimes \sigma_E)$.

Note that we chose the infimum and the supremum in a specific order in the definition of $\mathbb{D}^{\text{mx}}$, which, however, can be interchanged, as pointed out in the lemma below. This is one of the reasons we coined the term “minimax channel divergence” as the minimax equality holds for this definition under suitable conditions.

Lemma 6. (Minimax property.) Let $\mathbb{D}(\cdot \parallel \cdot)$ be a lower-semi-continuous quantum divergence with direct-sum property. It holds for any $\mathcal{N} \in \text{CPTP}(AE : B)$ and $\mathcal{M} \in \text{CP}(AE : B)$ that

$$\begin{aligned} \mathbb{D}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B} \parallel \mathcal{M}_{AE \rightarrow B}) &:= \sup_{\rho_{AR}} \inf_{\sigma_E, \omega_E} \mathbb{D}(\mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \parallel \mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \omega_E)) \quad (11) \\ &= \sup_{\rho_A} \inf_{\sigma_E, \omega_E} \mathbb{D}(\mathcal{N}_{AE \rightarrow B}(|\rho\rangle\langle\rho|_{AR} \otimes \sigma_E) \parallel \mathcal{M}_{AE \rightarrow B}(|\rho\rangle\langle\rho|_{AR} \otimes \omega_E)) \quad (12) \\ &= \inf_{\sigma_E, \omega_E} \sup_{\rho_A} \mathbb{D}(\mathcal{N}_{AE \rightarrow B}(|\rho\rangle\langle\rho|_{AR} \otimes \sigma_E) \parallel \mathcal{M}_{AE \rightarrow B}(|\rho\rangle\langle\rho|_{AR} \otimes \omega_E)) \quad (13) \\ &= \inf_{\sigma_E, \omega_E} \sup_{\rho_{AR}} \mathbb{D}(\mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \parallel \mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \omega_E)) \quad (14) \end{aligned}$$

where $|\rho\rangle_{AR}$ is a purification of ρ_A with $\mathcal{H}_R = \mathcal{H}_A$.

Proof. For any $\mathcal{C}_{AE \rightarrow B} \in \text{CP}(AE : B)$, let $\mathcal{C}_{A \rightarrow B}^{\varrho_E}$ denotes the induced channel $\mathcal{C}_{A \rightarrow B}^{\varrho_E}(\rho_A) := \mathcal{C}_{AE \rightarrow B}(\rho_A \otimes \varrho_E)$. For any fixed $\sigma_E, \omega_E \in \mathcal{D}(\mathcal{H}_E)$, we can write

$$\begin{aligned} &\sup_{\rho_{AR} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_R)} \mathbb{D}(\mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \parallel \mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \omega_E)) \\ &= \sup_{\rho_{AR} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_R)} \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\sigma_E}(\rho_{AR}) \parallel \mathcal{M}_{A \rightarrow B}^{\omega_E}(\rho_{AR})). \quad (15) \end{aligned}$$

As a consequence of purification, data processing and the Schmidt decomposition, we can restrict ρ_{AR} to pure states, and restrict the system R to be isomorphic to the system A, i.e.,

$$\begin{aligned} & \sup_{\rho_{AR} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_R)} \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\sigma_E}(\rho_{AR}) \| \mathcal{M}_{A \rightarrow B}^{\omega_E}(\rho_{AR})) \\ &= \sup_{|\phi\rangle \in \mathcal{H}_{AR}} \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\sigma_E}(|\phi\rangle\langle\phi|_{AR}) \| \mathcal{M}_{A \rightarrow B}^{\omega_E}(|\phi\rangle\langle\phi|_{AR})) \end{aligned} \quad (16)$$

$$= \sup_{\rho_A \in \mathcal{D}(\mathcal{H}_A)} \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\sigma_E}(|\rho\rangle\langle\rho|_{AR}) \| \mathcal{M}_{A \rightarrow B}^{\omega_E}(|\rho\rangle\langle\rho|_{AR})), \quad (17)$$

where the second line follows from the isometric invariance of quantum divergence. This implies that (13) equals (14). By [41, Lemma II.3], the direct-sum property implies that the objective function $\mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\sigma_E}(|\rho\rangle\langle\rho|_{AR} \otimes \sigma_E) \| \mathcal{M}_{A \rightarrow B}^{\omega_E}(|\rho\rangle\langle\rho|_{AR} \otimes \omega_E))$ is concave in the marginal state ρ_A . Furthermore, the combination of the direct-sum property and data processing ensures convexity in σ_E and ω_E . Therefore, by the minimax theorem [42], we can interchange the order of the supremum and infimum, so that (12) equals (13). It is also clear that (14) is no smaller than (11), which in turn is no smaller than (12). This completes the proof. $\square$

Remark 7. Lemma 6 allows us to express the minimax channel divergence in terms of the best-case channel divergence as

$$\mathbb{D}^{\text{mx}}(\mathcal{N}_{A \rightarrow B} \| \mathcal{M}_{A \rightarrow B}) = \inf_{\sigma_E, \omega_E} \mathbb{D}^\uparrow(\mathcal{N}_{A \rightarrow B}^{\sigma_E} \| \mathcal{M}_{A \rightarrow B}^{\omega_E}). \quad (18)$$

In the following, we show an interesting property of the minimax channel divergence when applied to n -fold tensor product channels. In particular, we prove that the infimum in Eq. (13) can be restricted to permutation-invariant states without changing the value of n -fold $\mathbb{D}^{\text{mx}}$.

Lemma 8. (Symmetry reduction.) Let $\mathbb{D}(\cdot \| \cdot)$ be a lower-semi-continuous quantum divergence with direct-sum property. Let A, B and E be quantum systems. For any $\mathcal{N} \in \text{CPTP}(A : B)$ and $\mathcal{M} \in \text{CP}(A : B)$, and positive integer $n \geq 2$,

$$\mathbb{D}^{\text{mx}}(\mathcal{N}_{A \rightarrow B}^{\otimes n} \| \mathcal{M}_{A \rightarrow B}^{\otimes n}) = \inf_{\sigma_{E_1^n}, \omega_{E_1^n} \in \text{PERM}(\mathcal{H}_{E_1^n})} \sup_{\rho_{A_1^n} \in \mathcal{D}(\mathcal{H}_{A_1^n})} \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \sigma_{E_1^n}) \| \mathcal{M}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \omega_{E_1^n})) \quad (19)$$

$$= \inf_{\sigma_{E_1^n}, \omega_{E_1^n} \in \text{PERM}(\mathcal{H}_{E_1^n})} \sup_{\rho_{A_1^n} \in \text{PERM}(\mathcal{H}_{A_1^n})} \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \sigma_{E_1^n}) \| \mathcal{M}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \omega_{E_1^n})), \quad (20)$$

where $|\rho\rangle_{AR}$ is a purification of ρ_A with $\mathcal{H}_R = \mathcal{H}_A$, and where $\text{PERM}(\mathcal{H}^{\otimes n})$ denote the set of all permutation invariant states on $\mathcal{H}^{\otimes n}$.

Proof. The key is to prove the following expression, as a function of $\sigma_{E_1^n}$ and $\omega_{E_1^n}$,

$$\begin{aligned} f_n(\sigma_{E_1^n}, \omega_{E_1^n}) &:= \sup_{\rho_{A_1^n} \in \mathcal{D}(\mathcal{H}_{A_1^n})} \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \sigma_{E_1^n}) \| \mathcal{M}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \omega_{E_1^n})) \\ &=: f_n(\sigma_{E_1^n}, \omega_{E_1^n} | \rho_{A_1^n}) \end{aligned} \quad (21)$$

to be permutation invariant. For each permutation π in the permutation group $\mathcal{S}_n$, we define the unitary transformation $\pi_{A_1^n} \in \mathcal{L}(\mathcal{H}_{A_1^n}^{\otimes n})$ as

$$\pi_{A_1^n} : |\psi_1\rangle_{A_1} \cdots |\psi_n\rangle_{A_n} \mapsto |\psi_{\pi(1)}\rangle_{A_1} \cdots |\psi_{\pi(n)}\rangle_{A_n}. \quad (22)$$

We also abuse the notation a bit, and use $\pi_{A_1^n}$ to denote the CPTP $\pi_{A_1^n} : \rho_{A_1^n} \mapsto \pi_{A_1^n} \rho_{A_1^n} \pi_{A_1^n}^\dagger$ as well. By carrying over the permutation of the output systems of the tensor of quantum channel to its input system (cf. [38, Eq. (2)] and [39, Theorem 3.3]), we have

$$\begin{aligned} & \pi_{B_1^n}(\mathcal{N}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \sigma_{E_1^n})) \\ &= \mathcal{N}_{A \rightarrow B}^{\otimes n}(\pi_{A_1^n E_1^n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \sigma_{E_1^n})) \end{aligned} \quad (23)$$

$$= \mathcal{N}_{A \rightarrow B}^{\otimes n}(\pi_{A_1^n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n}) \otimes \pi_{E_1^n}(\sigma_{E_1^n})) \quad (24)$$

$$= \mathcal{N}_{A \rightarrow B}^{\otimes n}(|\pi_{A_1^n} \rho\rangle\langle\pi_{A_1^n} \rho|_{A_1^n R_1^n} \otimes \pi_{E_1^n}(\sigma_{E_1^n})). \quad (25)$$

Similarly, the above equalities also hold for the channel $\mathcal{M}$. Since, quantum divergences are invariant under unitary transformations, we have

$$\begin{aligned} & f_n(\sigma_{E_1^n}, \omega_{E_1^n} | \rho_{A_1^n}) \\ &= \mathbb{D}(\pi_{B_1^n}(\mathcal{N}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \sigma_{E_1^n})) \| \pi_{B_1^n}(\mathcal{M}_{A \rightarrow B}^{\otimes n}(|\rho\rangle\langle\rho|_{A_1^n R_1^n} \otimes \omega_{E_1^n}))) \end{aligned} \quad (26)$$

$$= \mathbb{D}(\mathcal{N}_{A \rightarrow B}^{\otimes n}(|\pi_{A_1^n} \rho\rangle\langle\pi_{A_1^n} \rho|_{A_1^n R_1^n} \otimes \pi_{E_1^n}(\sigma_{E_1^n})) \| \mathcal{M}_{A \rightarrow B}^{\otimes n}(|\pi_{A_1^n} \rho\rangle\langle\pi_{A_1^n} \rho|_{A_1^n R_1^n} \otimes \pi_{E_1^n}(\omega_{E_1^n}))) \quad (27)$$

$$= f_n(\pi_{E_1^n}(\sigma_{E_1^n}), \pi_{E_1^n}(\omega_{E_1^n}) | \pi_{E_1^n}(\rho_{A_1^n})). \quad (28)$$

Note that the set of all density operators on systems A_1^n is permutation invariant. Hence,

$$\begin{aligned} & f_n(\pi_{E_1^n}(\sigma_{E_1^n}), \pi_{E_1^n}(\omega_{E_1^n})) \\ &= \sup_{\rho_{A_1^n} \in \mathcal{D}(\mathcal{H}_{A_1^n}^{\otimes n})} f_n(\pi_{E_1^n}(\sigma_{E_1^n}), \pi_{E_1^n}(\omega_{E_1^n}) | \rho_{A_1^n}) \end{aligned} \quad (29)$$

$$= \sup_{\rho_{A_1^n} \in \mathcal{D}(\mathcal{H}_{A_1^n}^{\otimes n})} f_n(\pi_{E_1^n}(\sigma_{E_1^n}), \pi_{E_1^n}(\omega_{E_1^n}) | \pi_{E_1^n}(\rho_{A_1^n})) \quad (30)$$

$$= \sup_{\rho_{A_1^n} \in \mathcal{D}(\mathcal{H}_{A_1^n}^{\otimes n})} f_n(\sigma_{E_1^n}, \omega_{E_1^n} | \rho_{A_1^n}), \quad (31)$$

i.e., f_n is permutation invariant.

Therefore, for any optimizing pair $(\sigma_{E_1^n}^*, \omega_{E_1^n}^*)$, we can replace them by the average over all of its permutations, i.e.,

$$(\sigma_{E_1^n}^{\text{new}}, \omega_{E_1^n}^{\text{new}}) := \left(\sum_{\pi \in \mathcal{S}_n} \frac{1}{|\mathcal{S}_n|} \pi_{E_1^n}(\sigma_{E_1^n}^*), \sum_{\pi \in \mathcal{S}_n} \frac{1}{|\mathcal{S}_n|} \pi_{E_1^n}(\omega_{E_1^n}^*) \right) \quad (32)$$

which is a pair of permutation invariant states; hence, finishing the proof of Eq. (19). Since the induced channels $\mathcal{N}_{A \rightarrow B}^{\otimes n}(\cdot \otimes \sigma_{E_1^n})$ and $\mathcal{M}_{A \rightarrow B}^{\otimes n}(\cdot \otimes \omega_{E_1^n})$ are permutation invariant for $\sigma_{E_1^n}, \omega_{E_1^n} \in \text{PERM}(\mathcal{H}_{E_1^n}^{\otimes n})$, we have Eq. (20) by applying [41, Proposition II.4]. $\square$

In the remainder of this section, we focuses on specific choices of $\mathbb{D}$ for further exploration of the properties of $\mathbb{D}^{\text{mx}}$, including Umegaki divergence $D(\rho \| \sigma) := \text{tr}[\rho(\log \rho - \log \sigma)]$ if $\rho \ll \sigma$ and $+\infty$ otherwise; measured divergence [43], [44] $D_M(\rho \| \sigma) := \sup_{\mathcal{X}, M} D(P_{\rho, M} \| P_{\sigma, M})$ where the optimization is over finite sets $\mathcal{X}$ and positive operator valued measures M on $\mathcal{X}$ such that $M_x \geq 0$ and $\sum_{x \in \mathcal{X}} M_x = I$, $P_{\rho, M}$ is a measure on $\mathcal{X}$ defined via the relation $P_{\rho, M}(x) = \text{tr}[M_x \rho]$ for any

$x \in \mathcal{X}$; and measured Rényi divergence [45] $D_{M,\alpha}(\rho\|\sigma) := \sup_{(\mathcal{X},M)} D_\alpha(P_{\rho,M}\|P_{\sigma,M})$ where D_α is the classical Rényi divergence.

Lemma 9. (*Super-additivity.*) Let $\alpha \in (0, +\infty)$. For any $\mathcal{N}_{AE \rightarrow B}^{(i)} \in \text{CPTP}(A_i E_i : B_i)$ and $\mathcal{M}_{AE \rightarrow B}^{(i)} \in \text{CP}(A_i E_i : B_i)$ with $i \in \{1, 2\}$,

$$D_{M,\alpha}^{\text{mx}} \left(\bigotimes_{i=1,2} \mathcal{N}_{A_i E_i \rightarrow B_i}^{(i)} \left\| \bigotimes_{i=1,2} \mathcal{M}_{A_i E_i \rightarrow B_i}^{(i)} \right\| \right) \geq \sum_{i=1,2} D_{M,\alpha}^{\text{mx}} \left(\mathcal{N}_{A_i E_i \rightarrow B_i}^{(i)} \left\| \mathcal{M}_{A_i E_i \rightarrow B_i}^{(i)} \right\| \right). \quad (33)$$

Proof. This is a direct result of Lemma 6 and the chain rule of the measured channel divergence [25, Lemma 2]. A complete proof can be found in the full paper [1]. $\square$

Remark 10. As an immediate consequence of Lemma 9, we have

$$D_{M,\alpha}^{\text{mx},\infty}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}) := \lim_{n \rightarrow \infty} \frac{1}{n} D_{M,\alpha}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B}^{\otimes n} \|\mathcal{M}_{AE \rightarrow B}^{\otimes n}) \\ = \sup_{n \in \mathbb{N}} \frac{1}{n} D_{M,\alpha}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B}^{\otimes n} \|\mathcal{M}_{AE \rightarrow B}^{\otimes n}), \quad (34)$$

where the second equality follows from Fekete's lemma.

Lemma 11. (*One-shot continuity.*) For any $\mathcal{N} \in \text{CPTP}(A : B)$ and $\mathcal{M} \in \text{CP}(A : B)$,

$$\sup_{\alpha \in (0,1)} D_{M,\alpha}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}) = D_M^{\text{mx}}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}). \quad (35)$$

Proof. This is a direct result of the continuity of the worst-case channel divergence [46, Lemma 22]. A complete proof can be found in the full paper [1]. $\square$

Lemma 12. (*Asymptotic continuity.*) For any $\mathcal{N} \in \text{CPTP}(A : B)$ and $\mathcal{M} \in \text{CP}(A : B)$,

$$\sup_{\alpha \in (0,1)} D_{M,\alpha}^{\text{mx},\infty}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}) \\ = D_M^{\text{mx},\infty}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}). \quad (36)$$

Proof. The proof is done via Remark 10 and Lemma 11 as follows.

$$\text{LHS of (36)} = \sup_{\alpha \in (0,1)} \sup_{n \in \mathbb{N}} \frac{1}{n} D_{M,\alpha}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B}^{\otimes n} \|\mathcal{M}_{AE \rightarrow B}^{\otimes n}) \quad (37)$$

$$= \sup_{n \in \mathbb{N}} \frac{1}{n} \sup_{\alpha \in (0,1)} D_{M,\alpha}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B}^{\otimes n} \|\mathcal{M}_{AE \rightarrow B}^{\otimes n}) \quad (38)$$

$$= \sup_{n \in \mathbb{N}} \frac{1}{n} D_M^{\text{mx}}(\mathcal{N}_{AE \rightarrow B}^{\otimes n} \|\mathcal{M}_{AE \rightarrow B}^{\otimes n}) = \text{RHS of (36)}. \quad \square$$

To finish this section, we establish the asymptotic equivalence of the measured minimax channel divergence and the Umegaki minimax channel divergence as follows.

Lemma 13. (*Asymptotic equivalence.*) For any $\mathcal{N} \in \text{CPTP}(A : B)$ and $\mathcal{M} \in \text{CP}(A : B)$,

$$D_M^{\text{mx},\infty}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}) = D^{\text{mx},\infty}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}). \quad (39)$$

Proof. Since that $D_M(\cdot\|\cdot) \leq D(\cdot\|\cdot)$ in general, it is clear that $\text{LHS} \leq \text{RHS}$; and the other direction is a direct result of Remark 10, Lemma 11, alongside [47, Lemma A.1] and [46, Lemma 16]. We omit the details. $\square$

IV. QUANTUM CHANNEL DISCRIMINATION AGAINST JAMMERS

We are interested in the task of channel discrimination between two channels with jammers $\mathcal{N}_{AE \rightarrow B}, \mathcal{M}_{AE \rightarrow B} \in \text{CPTP}(A : B)$, where A and B are the input and the output system, respectively, and E is the adversary/jammer system. The type-I and type-II error are described by

$$\alpha(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}) := \sup_{\sigma_E \in \mathcal{D}(\mathcal{H}_E)} \text{tr}(\mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \cdot (I_B - M)), \quad (40)$$

$$\beta(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}) := \sup_{\sigma_E \in \mathcal{D}(\mathcal{H}_E)} \text{tr}(\mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \cdot M), \quad (41)$$

respectively, where input preparation $\rho_{AR} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_R)$ and measurement $0 \preceq M \preceq I_B$ on the output system fully describes the discriminating strategy by the tester, and where R is some auxiliary system. The optimal type-II error probability is defined as

$$\beta_\epsilon^E(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}) := \inf_{0 \preceq M \preceq I_B, \rho_{AR} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_R)} \left\{ \beta(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}) \mid \alpha(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}) \leq \epsilon \right\}. \quad (42)$$

and we are interested in the error exponent

$$\lim_{n \rightarrow \infty} -\frac{1}{n} \log \beta_\epsilon^E(\mathcal{N}_{AE \rightarrow B}^{\otimes n} \|\mathcal{M}_{AE \rightarrow B}^{\otimes n}) = ? \quad (2)$$

We link the operational quantity as in (42) to the minimax channel divergence as in the following proposition.

Proposition 14. For any $\mathcal{N} \in \text{CPTP}(A : B)$ and $\mathcal{M} \in \text{CP}(A : B)$,

$$-\log \beta_\epsilon^E(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}) = D_{H,\epsilon}^{\text{mx}}(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}). \quad (43)$$

Here, for each $\epsilon \in [0, 1]$, $D_{H,\epsilon}(\rho\|\sigma) := -\log \beta_\epsilon(\rho\|\sigma)$ where $\beta_\epsilon(\rho\|\sigma) := \min_{0 \preceq M \preceq I} \{ \text{tr}[\sigma M] : \text{tr}[\rho(I - M)] \leq \epsilon \}$.

Proof. By definition 4, we can write $D_{H,\epsilon}^{\text{mx}}$ as

RHS of (43)

$$:= \sup_{\rho_{AR}} \inf_{\sigma_E, \omega_E} D_{H,\epsilon}(\mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \|\mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \omega_E)) \quad (44) \\ = -\log \inf_{\rho_{AR}} \sup_{\sigma_E, \omega_E} \beta_\epsilon(\mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \sigma_E) \|\mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \omega_E)). \quad (45)$$

Denote

$$\beta_\epsilon^E(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}) := \inf_{0 \preceq M \preceq I_B} \left\{ \beta(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}) \mid \alpha(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}) \leq \epsilon \right\}. \quad (46)$$

Immediately we have

$$\beta_\epsilon^E(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B}) = \inf_{\rho_{AR}} \beta_\epsilon^E(\mathcal{N}_{AE \rightarrow B} \|\mathcal{M}_{AE \rightarrow B} | \rho_{AR}). \quad (47)$$

Now, let $\mathcal{A}^{\rho_{AR}}$ and $\mathcal{B}^{\rho_{AR}}$ denote the image of the induced channels $\mathcal{N}_{E \rightarrow B}^{\rho_{AR}}$ and $\mathcal{M}_{E \rightarrow B}^{\rho_{AR}}$ (see Remark 5 for the notation of the induced channels), respectively, i.e.,

$$\mathcal{A}^{\rho_{AR}} := \{ \mathcal{N}_{AE \rightarrow B}(\rho_{AR} \otimes \varrho_E) \mid \varrho_E \in \mathcal{D}(\mathcal{H}_E) \}, \quad (48)$$

$$\mathcal{B}^{\rho_{AR}} := \{ \mathcal{M}_{AE \rightarrow B}(\rho_{AR} \otimes \varrho_E) \mid \varrho_E \in \mathcal{D}(\mathcal{H}_E) \}. \quad (49)$$

It is straightforward that these two sets are convex; and moreover, we have

$$\alpha(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}} | \rho_{\text{AR}}) = \alpha(\mathcal{A}^{\rho_{\text{AR}}}, M), \quad (50)$$

$$\beta(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}} | \rho_{\text{AR}}) = \beta(\mathcal{B}^{\rho_{\text{AR}}}, M), \quad (51)$$

where $\alpha(\mathcal{D}, M) := \sup_{\varrho \in \mathcal{D}} \text{tr}(\varrho \cdot (I - M))$, and $\beta(\mathcal{D}, M) := \sup_{\varrho \in \mathcal{D}} \text{tr}(\varrho \cdot M)$. Therefore,

$$\beta_{\epsilon}^{\text{E}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}} | \rho_{\text{AR}}) = \inf_{0 \leq M \leq I_{\text{B}}} \{ \beta(\mathcal{B}^{\rho_{\text{AR}}}, M) | \alpha(\mathcal{A}^{\rho_{\text{AR}}}, M) \leq \epsilon \} \quad (52)$$

$$= \sup_{\bar{\rho} \in \mathcal{A}^{\rho_{\text{AR}}}, \bar{\sigma} \in \mathcal{B}^{\rho_{\text{AR}}}} \beta_{\epsilon}(\bar{\rho} \| \bar{\sigma}) \quad (53)$$

$$= \sup_{\sigma_{\text{E}}, \omega_{\text{E}}} \beta_{\epsilon}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}(\rho_{\text{AR}} \otimes \sigma_{\text{E}}) \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}(\rho_{\text{AR}} \otimes \omega_{\text{E}})), \quad (54)$$

where (53) follows from [46, Lemma 31]. Combining with (47), we have

$$\begin{aligned} \beta_{\epsilon}^{\text{E}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) &= \inf_{\rho_{\text{AR}}} \sup_{\sigma_{\text{E}}, \omega_{\text{E}}} \\ \beta_{\epsilon}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}(\rho_{\text{AR}} \otimes \sigma_{\text{E}}) \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}(\rho_{\text{AR}} \otimes \omega_{\text{E}})). \end{aligned} \quad (55)$$

We have the asserted result by taking $-\log$ on both sides. $\square$

Proposition 14 connects the optimal type-II error to the hypothesis testing minimax channel divergence. Therefore, the error exponent of question can be expressed as

$$\begin{aligned} \lim_{n \rightarrow \infty} -\frac{1}{n} \log \beta_{\epsilon}^{\text{E}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}^{\otimes n} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}^{\otimes n}) \\ = \lim_{n \rightarrow \infty} -\frac{1}{n} D_{\text{H}, \epsilon}^{\text{mx}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}^{\otimes n} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}^{\otimes n}) \end{aligned} \quad (56)$$

A weak asymptotic characterization is shown in the theorem below.

Theorem 15. (Stein's lemma.) For any $\mathcal{N} \in \text{CPTP}(\text{AE} : \text{B})$ and $\mathcal{M} \in \text{CP}(\text{AE} : \text{B})$,

$$\begin{aligned} \lim_{\epsilon \rightarrow 0^+} \lim_{n \rightarrow \infty} -\frac{1}{n} \log \beta_{\epsilon}^{\text{E}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}^{\otimes n} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}^{\otimes n}) \\ = D^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}). \end{aligned} \quad (57)$$

Proof. By Proposition 14, it is equivalent to prove that

$$\begin{aligned} \lim_{\epsilon \rightarrow 0^+} \lim_{n \rightarrow \infty} -\frac{1}{n} D_{\text{H}, \epsilon}^{\text{mx}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}^{\otimes n} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}^{\otimes n}) \\ = D^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}). \end{aligned} \quad (58)$$

By [48, Lemma 5] (along with the data processing inequality of the Petz-Rényi divergence), we have for any $\alpha, \epsilon \in (0, 1)$,

$$D_{\text{M}, \alpha}(\cdot \| \cdot) + \frac{\alpha}{\alpha - 1} \log \frac{1}{\epsilon} \leq D_{\text{H}, \epsilon}(\cdot \| \cdot) \leq \frac{1}{1 - \epsilon} (D(\cdot \| \cdot) + h(\epsilon)). \quad (59)$$

Hence,

$$\begin{aligned} D_{\text{M}, \alpha}^{\text{mx}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) + \frac{\alpha}{\alpha - 1} \log \frac{1}{\epsilon} \\ \leq D_{\text{H}, \epsilon}^{\text{mx}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) \end{aligned} \quad (60)$$

$$\leq \frac{1}{1 - \epsilon} (D^{\text{mx}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) + h(\epsilon)). \quad (61)$$

Applying the above to n -fold channels and dividing by n , and taking $n \rightarrow \infty$, then $\epsilon \rightarrow 0$,

$$\begin{aligned} D_{\text{M}, \alpha}^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) \\ \leq \lim_{\epsilon \rightarrow 0^+} \lim_{n \rightarrow \infty} \frac{1}{n} D_{\text{H}, \epsilon}^{\text{mx}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}^{\otimes n} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}^{\otimes n}) \end{aligned} \quad (62)$$

$$\leq D^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}). \quad (63)$$

As this holds for any $\alpha \in (0, 1)$, we can take the supremum over α , and replace the left-most expression by

$$\begin{aligned} \sup_{\alpha \in (0, 1)} D_{\text{M}, \alpha}^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) \\ = D_{\text{M}}^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) = D^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) \end{aligned} \quad (64)$$

where the equalities follows from Lemma 12 and Lemma 13, respectively. Therefore, we have

$$\begin{aligned} D^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}) \\ \leq \lim_{\epsilon \rightarrow 0^+} \lim_{n \rightarrow \infty} \frac{1}{n} D_{\text{H}, \epsilon}^{\text{mx}}(\mathcal{N}_{\text{AE} \rightarrow \text{B}}^{\otimes n} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}^{\otimes n}) \end{aligned} \quad (65)$$

$$\leq D^{\text{mx}, \infty}(\mathcal{N}_{\text{AE} \rightarrow \text{B}} \| \mathcal{M}_{\text{AE} \rightarrow \text{B}}). \quad (66)$$

which concludes the asserted result. $\square$

Remark 16. Note that when $\dim \mathcal{H}_{\text{E}} = 1$, the above theorem recovers the best-case channel discrimination result in [27, Theorem 3]. When $\dim \mathcal{H}_{\text{A}} = 1$, it recovers a weak version¹ of the worst-case (adversarial) channel discrimination result in [25, Theorem 1] under non-adaptive strategies. As the strong converse property (i.e., the convergence without the dependence of ϵ) of the best-case channel discrimination is still open [12], it is also unclear whether the strong converse property holds in the minimax setting considered here.

V. CONCLUSION

In this work, we studied quantum channel discrimination in the presence of jammers, formulating the problem as a competitive minimax game between the tester and the adversary, each optimizing their respective strategies. We established an asymptotic characterization of the optimal type-II error exponent under parallel strategies, showing that it is governed by the minimax channel divergence introduced herein. We also explored fundamental properties of the minimax channel divergence, which can be of independent interest and lay the foundation for future exploration.

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¹The previous result in [25, Theorem 1] has the strong converse property.

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