arXiv: 2410.14547

arXiv: 1909.02540 (PRL) & 2010.11822 (PRXQ)

Surpassing the fundamental limits of distillation with catalysts

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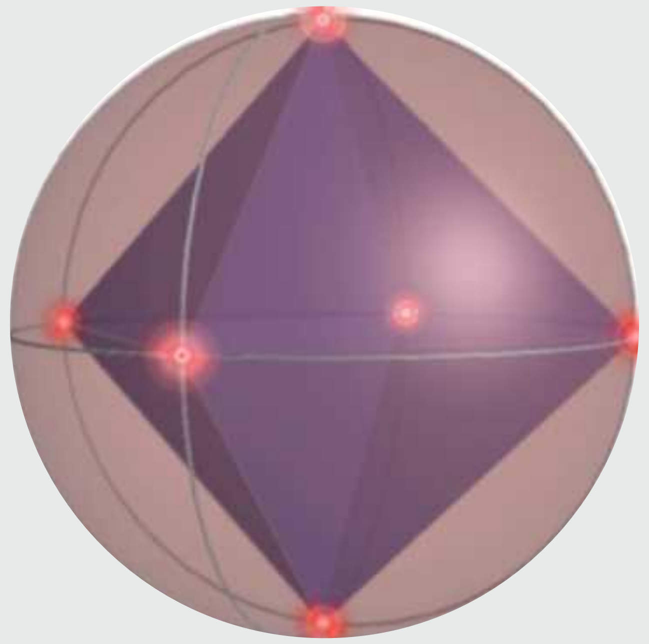
AQIS @ Hong Kong, August 2025

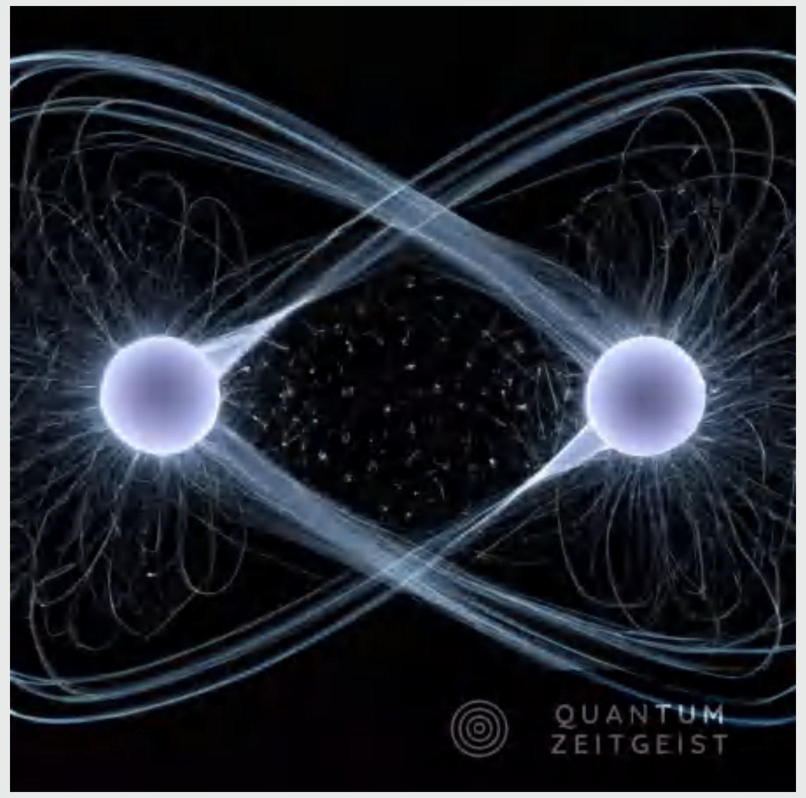
What makes quantum technologies powerful?

Quantum Resources [Chitambar-Gour-19, RMP]

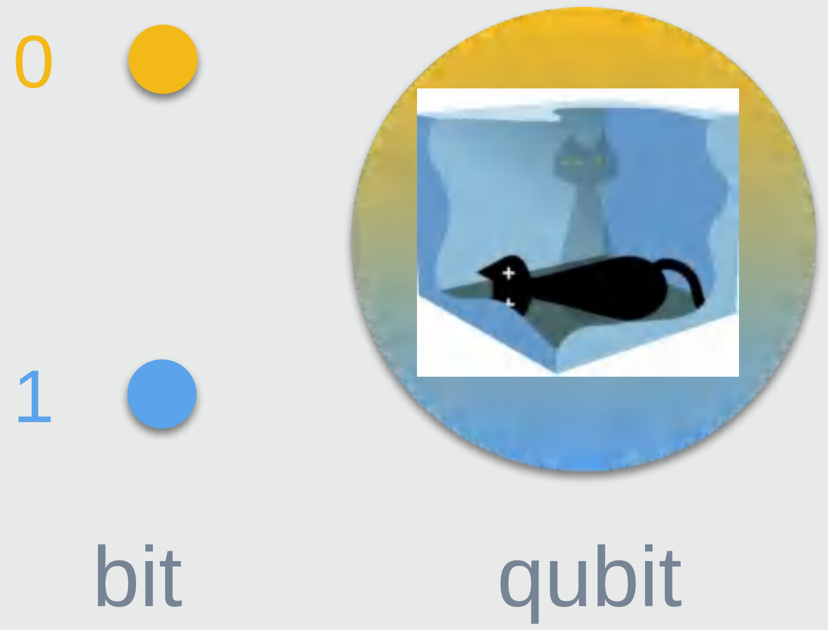
These resources serve as the key ingredient in quantum computing and quantum communication, just as oil is to a car.



Magic state



Quantum entanglement



Superposition/coherence

• • •

Quantum computation   Quantum communication   Quantum cryptography

What makes quantum technologies less powerful?

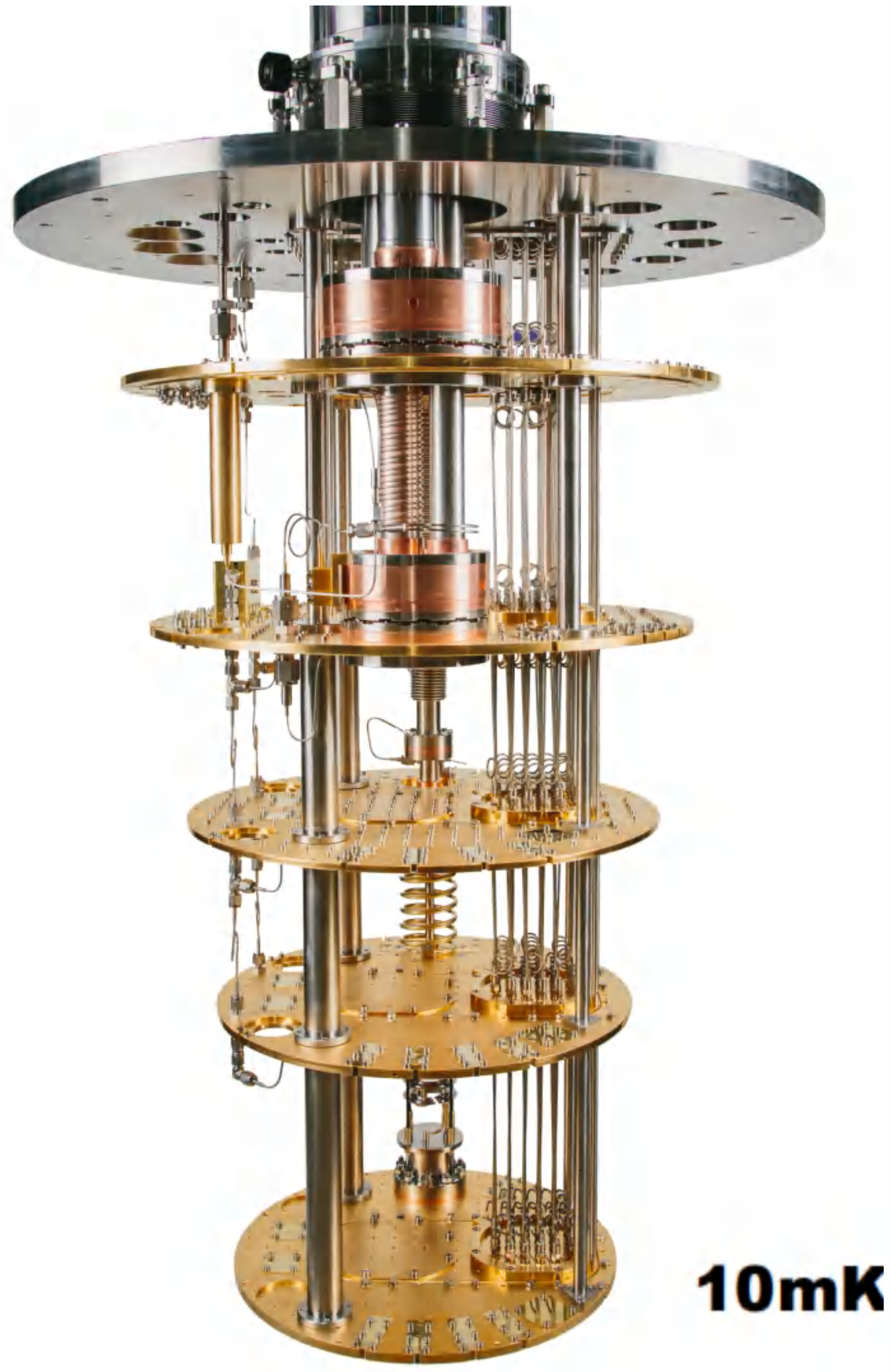
Quantum resources are fragile and highly susceptible to noise effects

- environmental noise
- imperfect controls
- unstable memories
- …

Unreliable for usage or lose power

Dilution refrigerator
cool down the temperature to near absolute zero

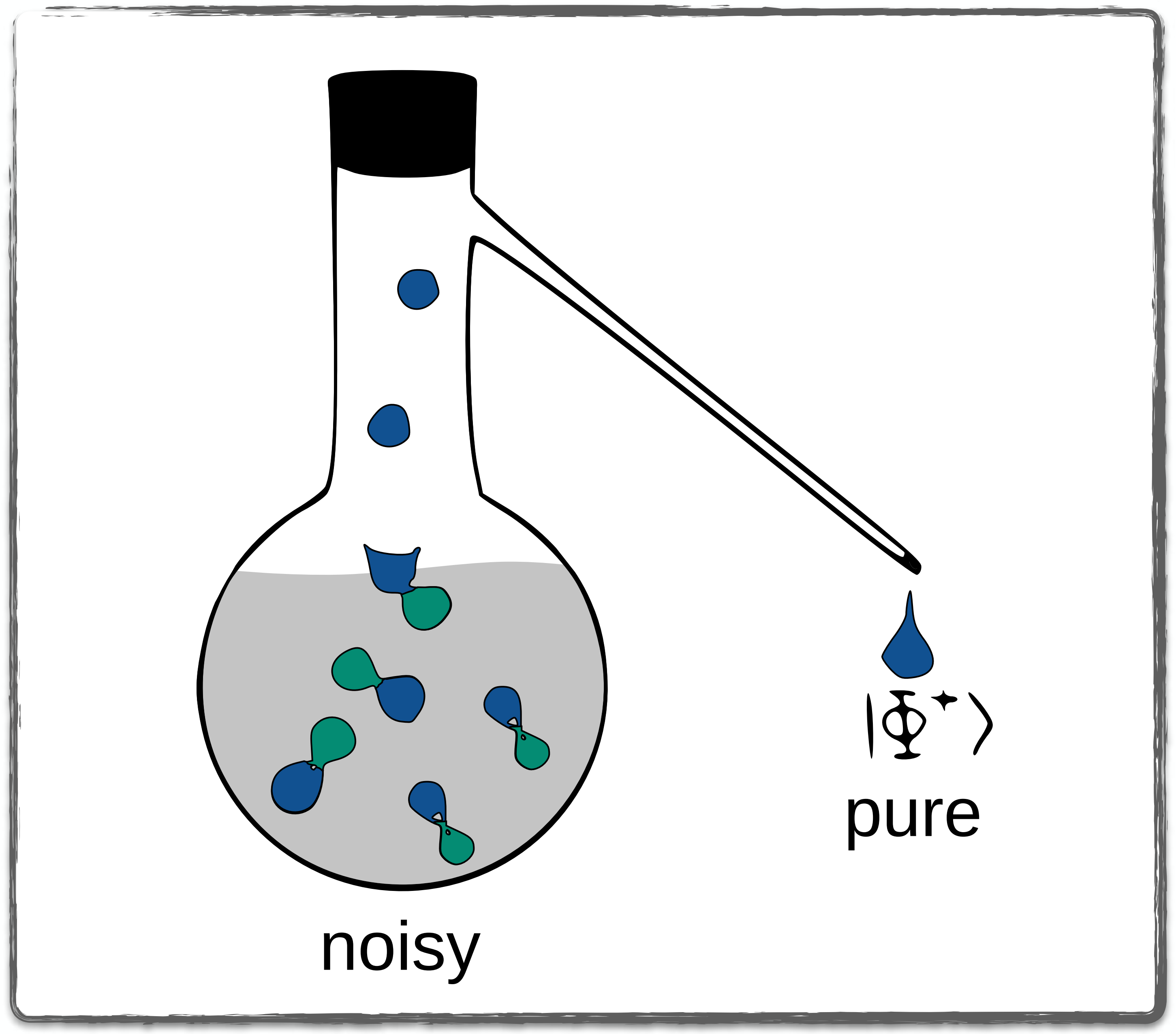
remove thermal noise and provide visibility to quantum behavior



10mK = -273.14 °C

Quantum resource distillation

A standard subroutine in QIS for overcoming noise



$$\rho^{\otimes n} \xrightarrow[\text{operations}]{\text{Constrained}} \psi^{\otimes m}$$

Noisy — Pure

Resource purification/distillation

[BBPSSW-96] quantum entanglement purification
[Bravyi-Kitaev-05] quantum magic distillation
[Aberg-06] quantum coherence distillation
…

1. **Possibility**: Is quantum resource purification possible?
2. **Distillable rate**: Given primitive n, find largest target m?
3. **Overhead**: Given target m, find smallest primitive n?

Quantum resource distillation

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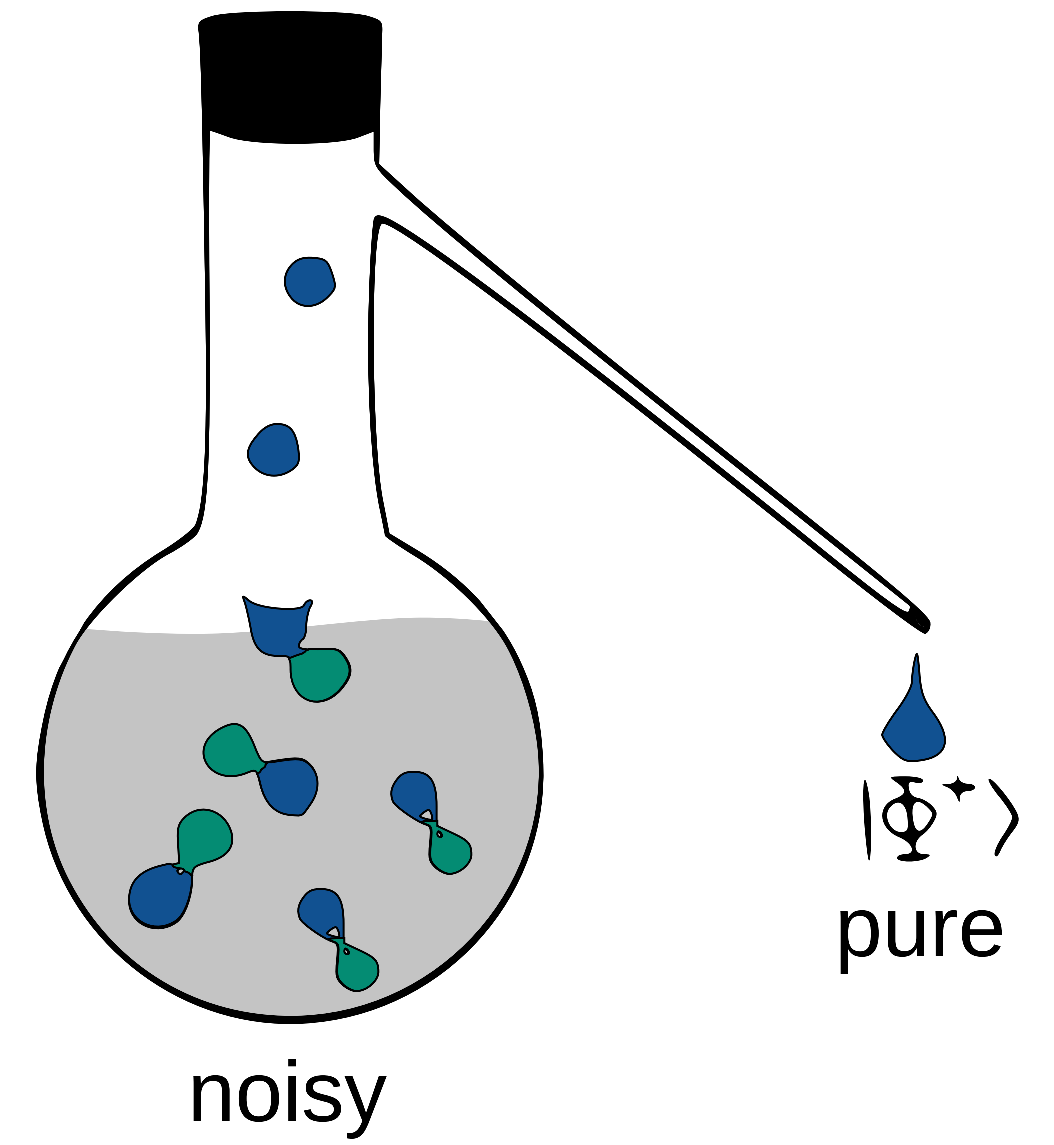

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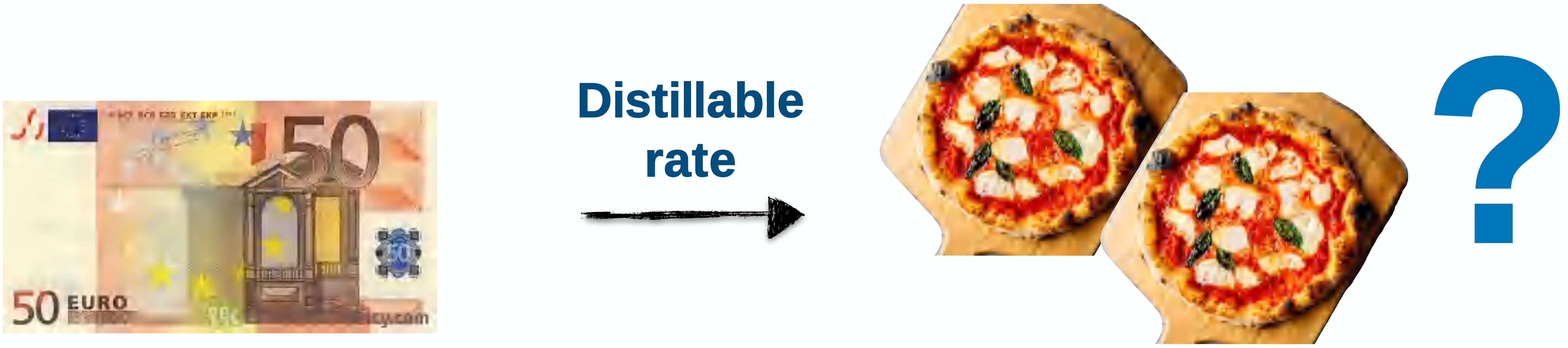

Quantum resource distillation

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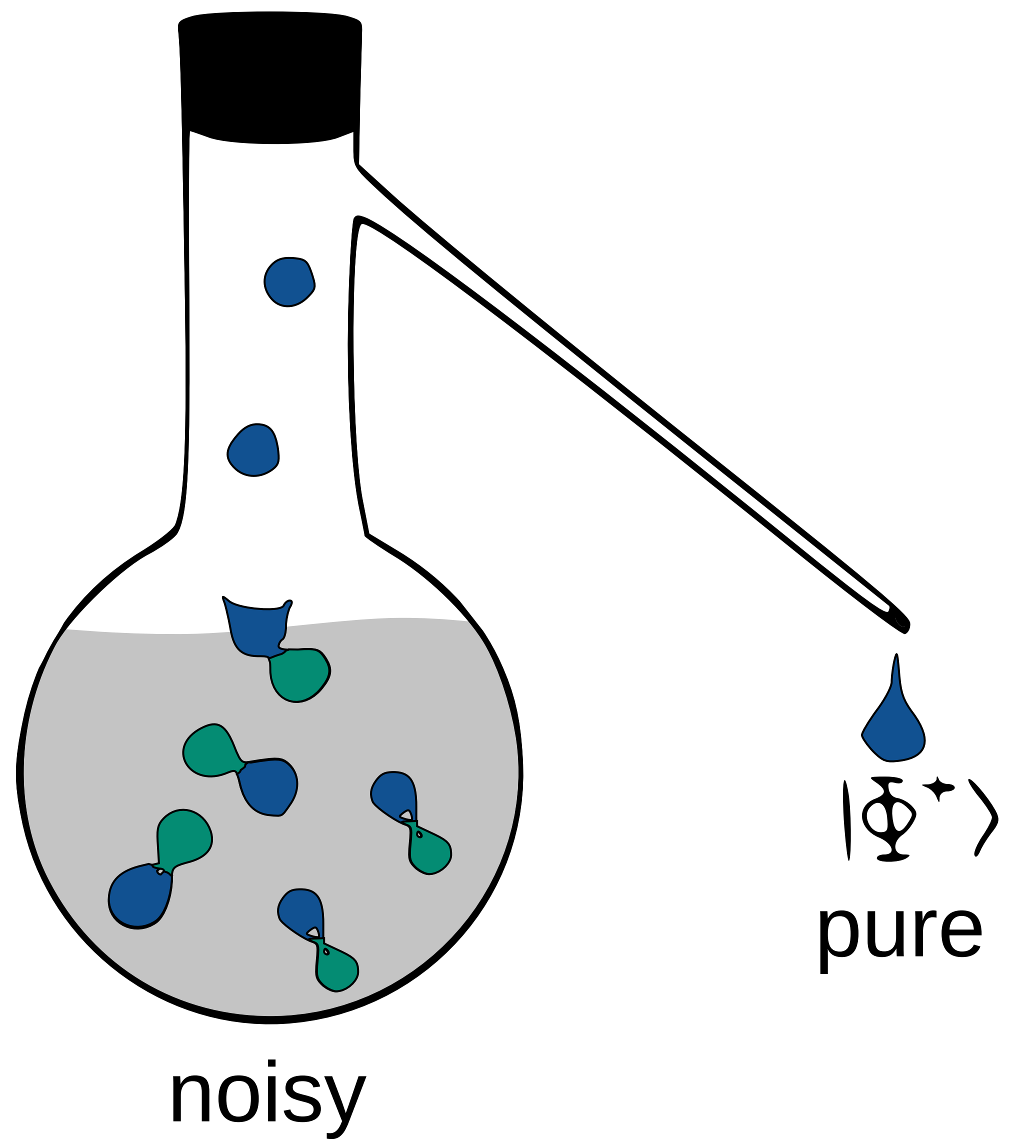

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Noisy | Pure

Resource purification/distillation

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1. **Possibility**: Is quantum resource purification possible?
2. **Distillable rate**: Given primitive n, find largest target m?
3. **Overhead**: Given target m, find smallest primitive n?

?

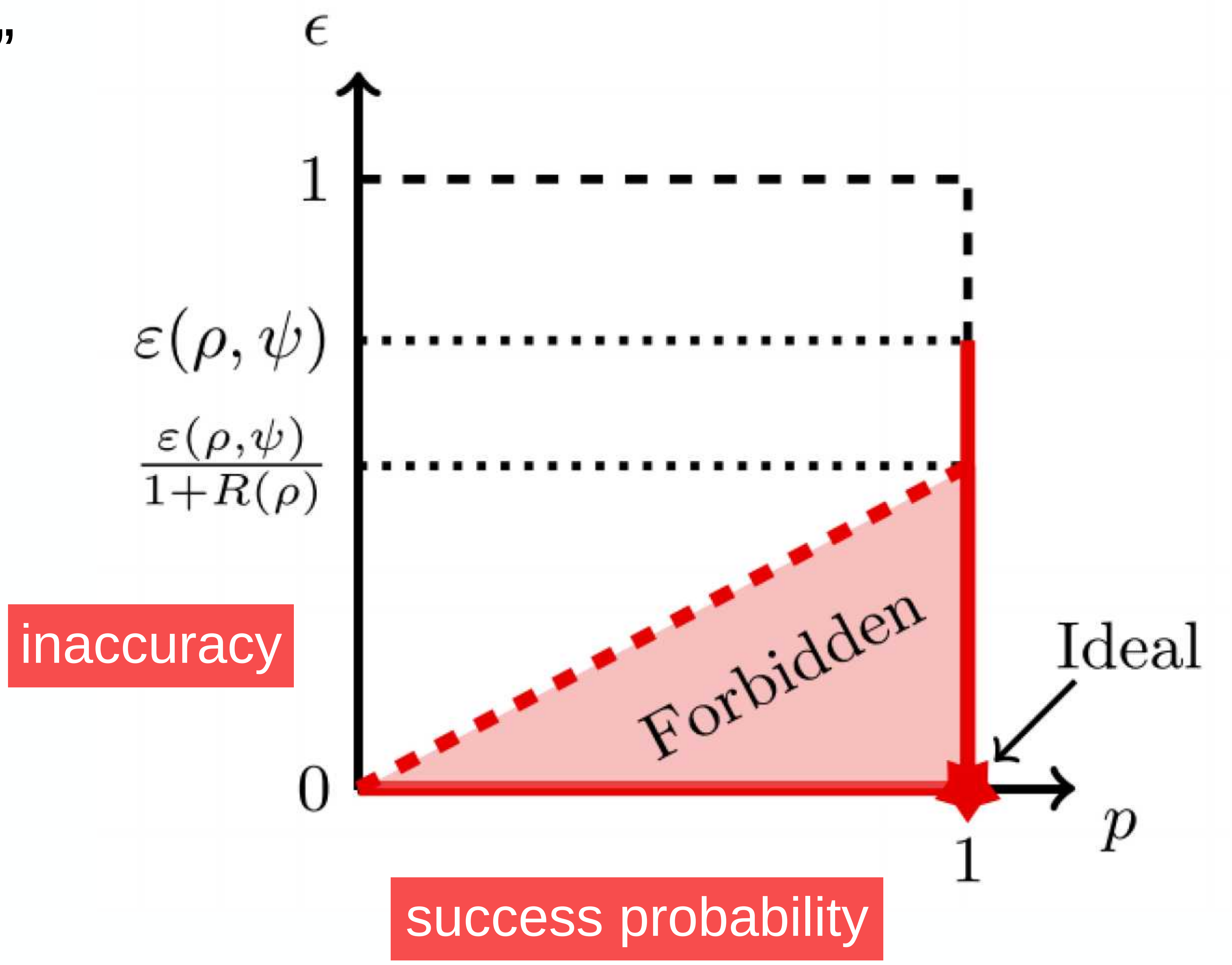
Overhead →

Fundamental limits of distillation

arXiv: 1909.02540 (PRL) & 2010.11822 (PRXQ)

Perfect resource purification is generically impossible, even probabilistically.

A trade-off bound akin to "uncertainty relation"



Universal law:

- Any well-defined resource theory
- Any pure target state
- Any free manipulation

$$\varepsilon(\rho,\psi) = \lambda_{\min}(\rho)(1 - f_\psi)$$

overlap with free states, always < 1

Application in magic state distillation

[Gottesman-Knill theorem]
classically simulable / fault-tolerant

[Bravyi-Kitaev-05]

Clifford gates + **Magic state** → Universal QC

A leading scheme for fault tolerant quantum computing

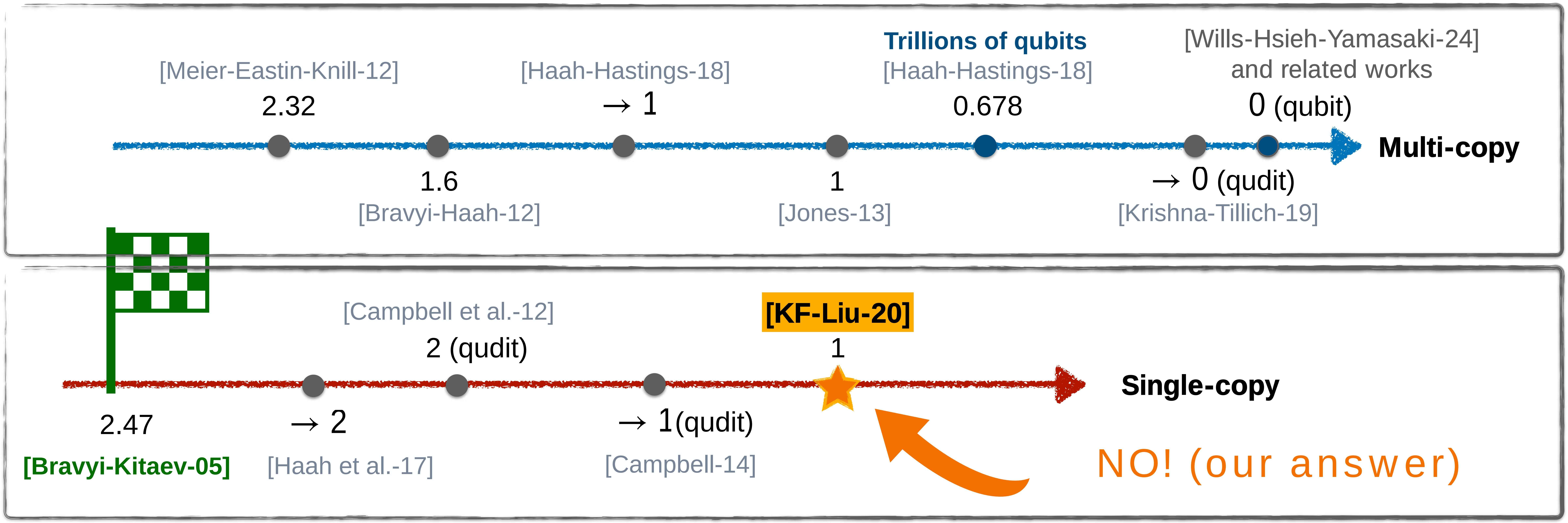
To ensure the computational accuracy, we need high-quality magic states.

Overhead ← $\rho^{\otimes n}$ $\xrightarrow[\text{operations}]{\text{Clifford}}_{\varepsilon}$ $|T\rangle$ magic state

Fundamental limits of magic state distillation

Overhead of magic state distillation $n \approx [\log(1/\varepsilon)]^{\gamma}$, where γ represents the resource cost

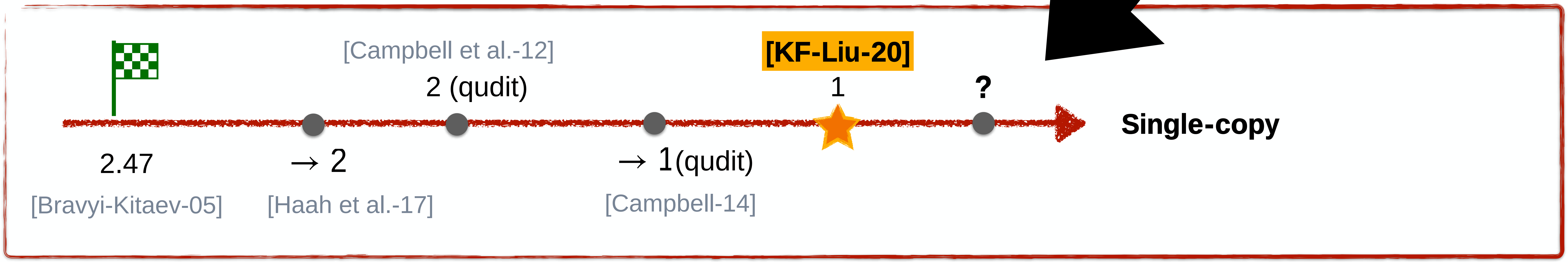
Open problem: can we distill one magic state with sub-logarithmic resources ($\gamma < 1$) ?



Is there a way to overcome this limit?

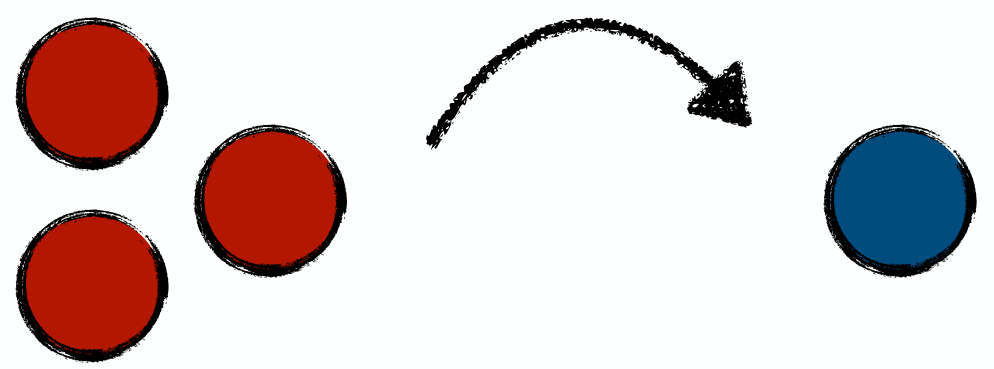
arXiv: 2410.14547

Yes, using catalyst!



Comparison of three distillation settings

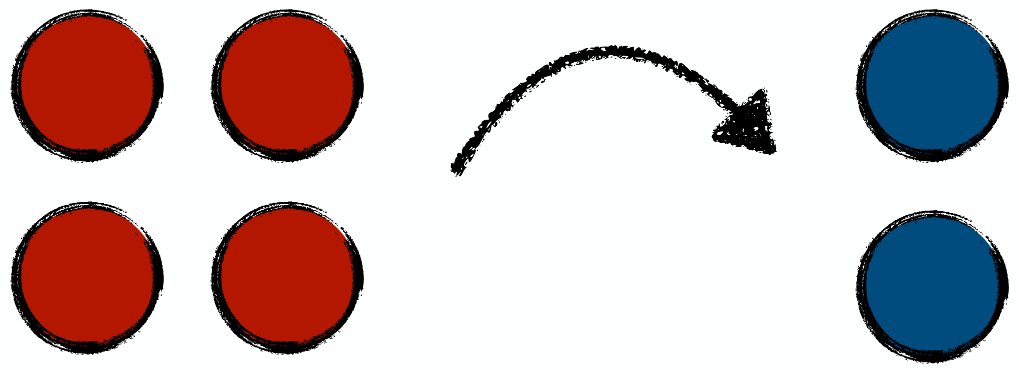
One-shot unassisted



$$C_{\varepsilon,p}(\rho,\sigma)$$

$$\min\left\{n : \rho^{\otimes n} \xrightarrow{(\varepsilon,p)} \sigma\right\}$$

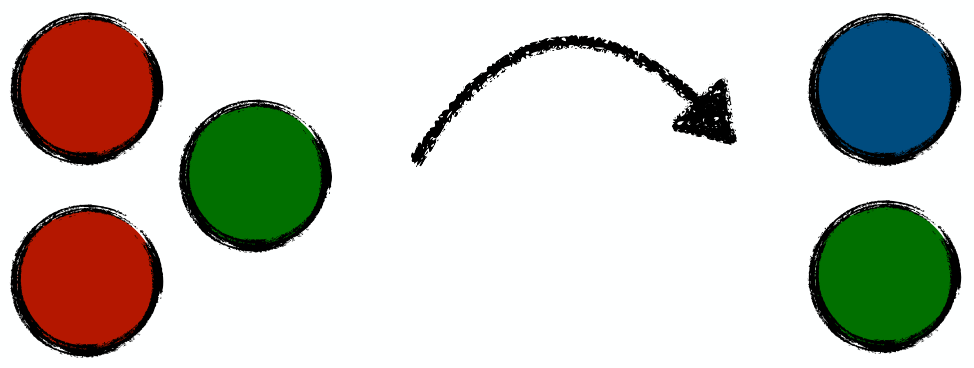
Multi-shot average



$$\bar{C}_{\varepsilon,p}(\rho,\sigma)$$

$$\min\left\{\lceil n/m \rceil : \rho^{\otimes n} \xrightarrow{(\varepsilon,p)} \sigma^{\otimes m}\right\}$$

One-shot catalytic



$$\tilde{C}_{\varepsilon,p}(\rho,\sigma)$$

$$\min\left\{n : \rho^{\otimes n} \otimes \omega \xrightarrow{(\varepsilon,p)} \sigma \otimes \omega\right\}$$

$\mathscr{L}(\rho^{\otimes n}) = p \cdot \eta^m$, with *trace distance* $\Delta(\eta_i^m, \sigma) \leq \varepsilon$, $\forall i$,
and η_i^m is the marginal of η^m

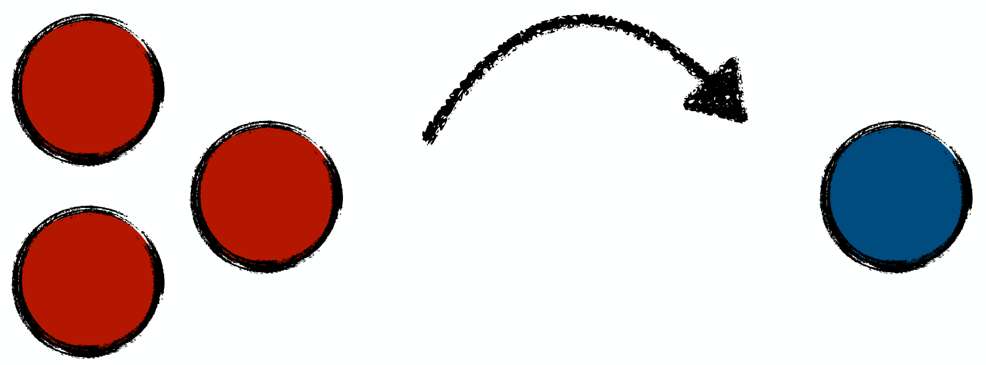
$\mathscr{L}(\rho_S^{\otimes n} \otimes \omega_A) = p \cdot \nu_{SA}$
with $\Delta(\nu_S, \sigma_S) \leq \varepsilon$ and $\nu_A = \omega_A$

With these notations, the no-go theorem sets the fundamental limit $C_{\varepsilon,p}(\rho,\sigma) = \Omega(\log(1/\varepsilon))$.

In the muti-shot average setting, m can scale with n. **One-shot settings align better with practice.**

Comparison of three distillation settings

One-shot unassisted



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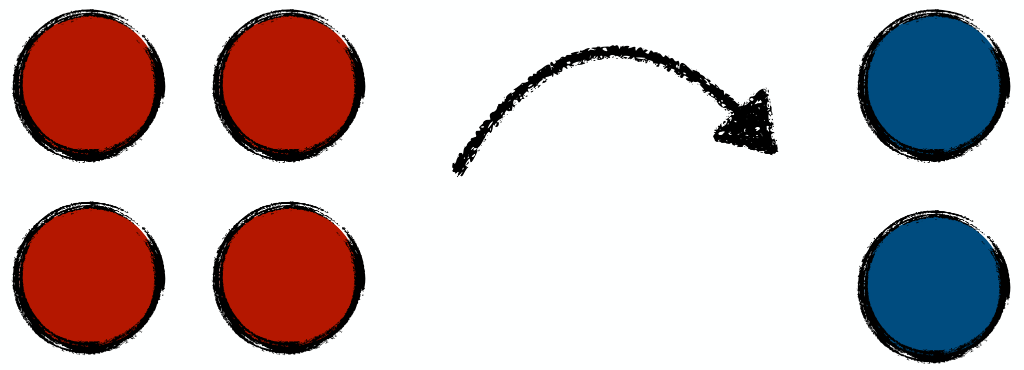

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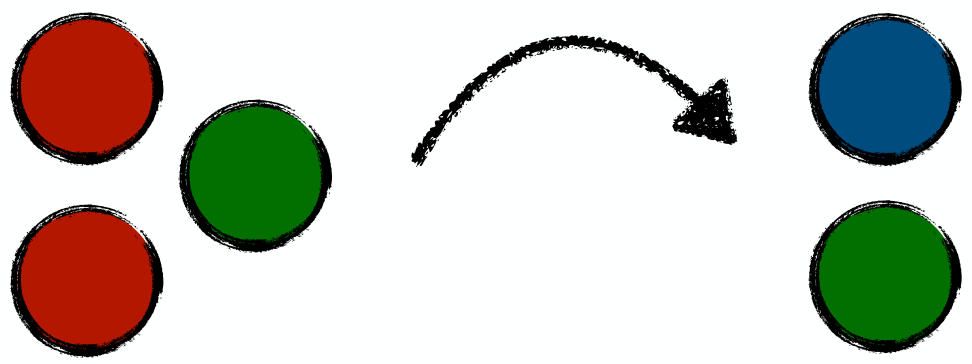
One-shot catalytic



$$\tilde{C}_{\varepsilon,p}(\rho,\sigma)$$

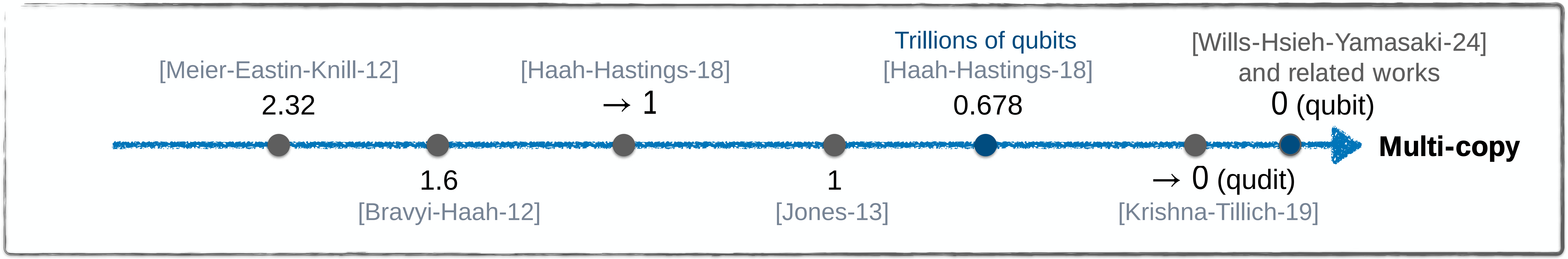
$$\min\left\{n : \rho^{\otimes n} \otimes \omega \xrightarrow{(\varepsilon,p)} \sigma \otimes \omega\right\}$$

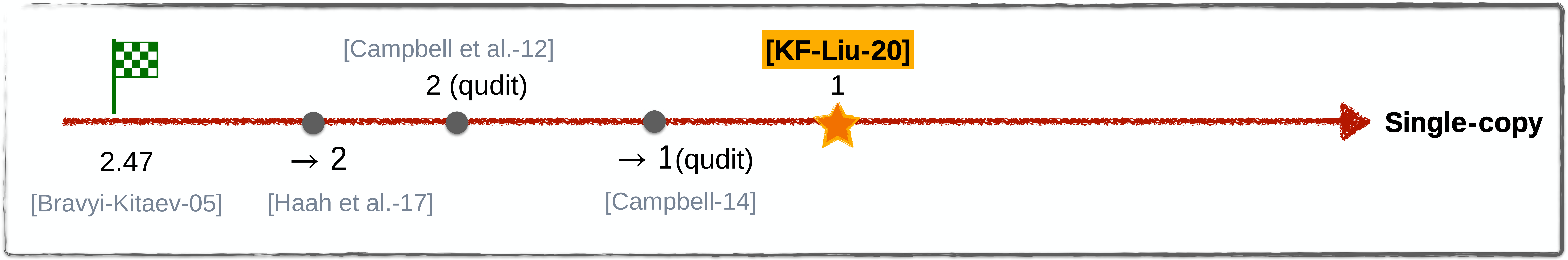
Our result

$$C_{\varepsilon,p}(\rho,\sigma) \geq \bar{C}_{\varepsilon,p}(\rho,\sigma) \geq \tilde{C}_{\varepsilon,p}(\rho,\sigma)$$

Main idea: any multi-shot distillation protocol from $\rho^{\otimes n}$ to $\sigma^{\otimes m}$ can effectively be turned into a one-shot catalytic distillation protocol from $\rho^{\lceil n/m \rceil} \otimes \omega$ to $\sigma \otimes \omega$ with the same performance.

Application in magic state distillation





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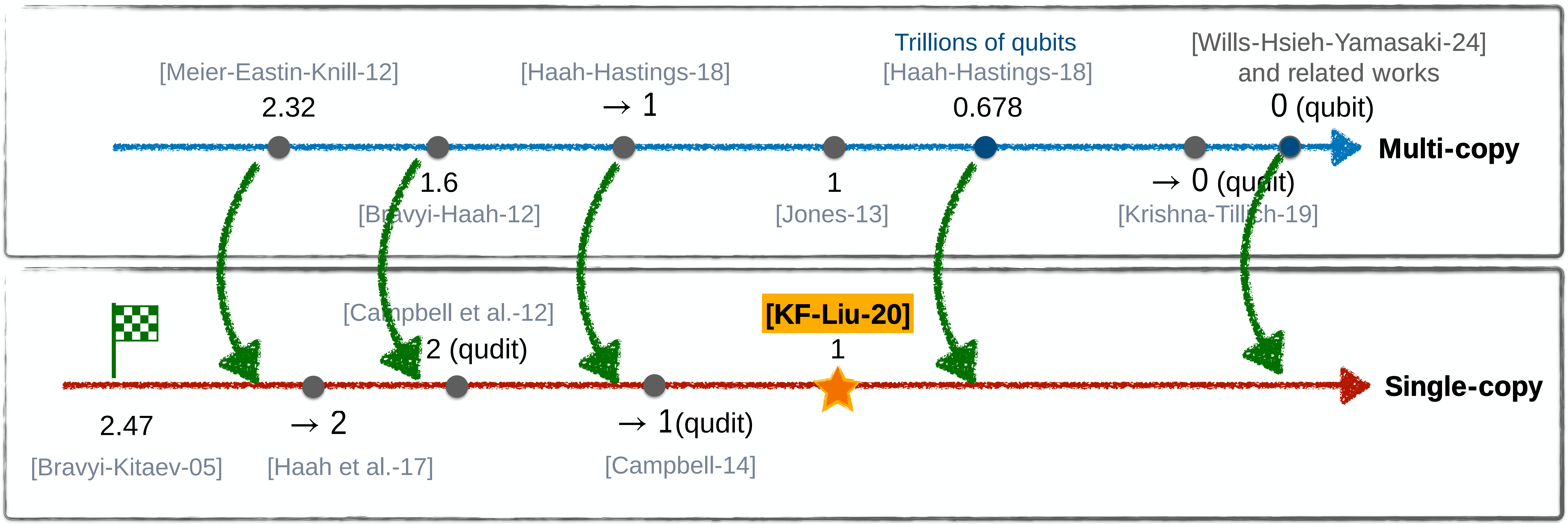

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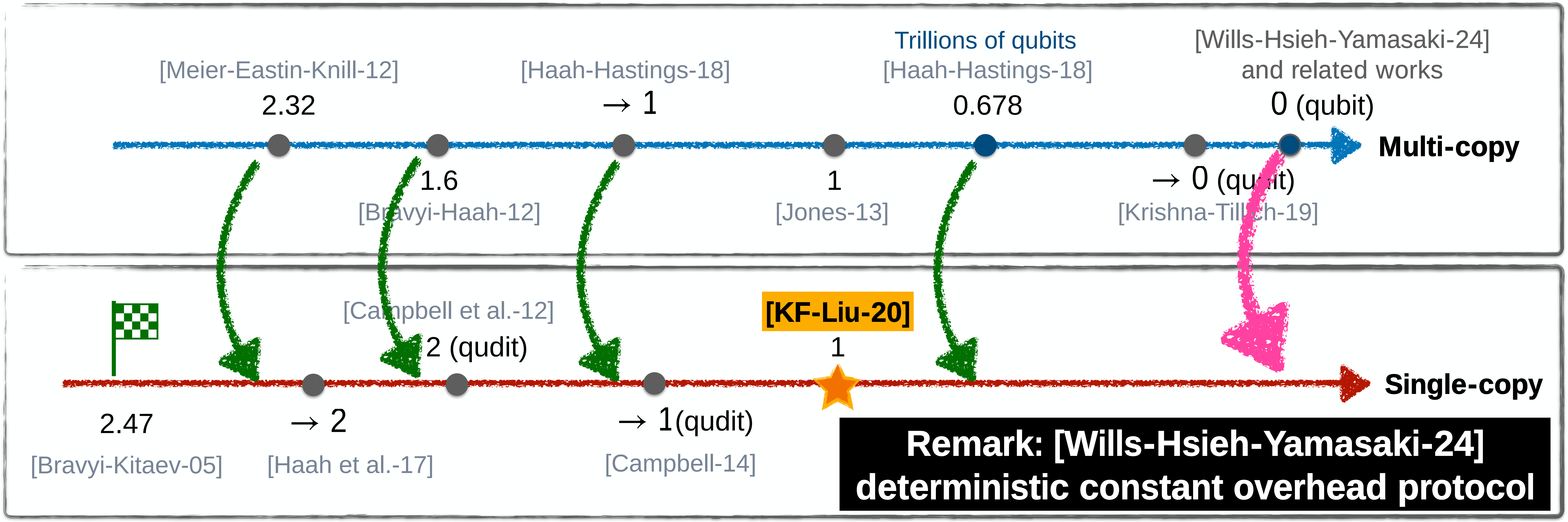


Our result

There exist **one-shot catalytic** magic state distillation protocols that achieve any given target error with unit success probability and **constant overhead**.

How large is this constant?

Not answered in [Wills-Hsieh-Yamasaki-24] and related works

Our answer: **ONE** (with catalyst)

Our result

There exist **one-shot catalytic** magic state distillation protocols that achieve any given target error with unit success probability and **constant overhead**.

Trading success probability for reduced overhead

(time) **(space)**

$$\rho^{\otimes n} \xrightarrow{(\varepsilon, p)} \sigma^{\otimes m} \quad \Longrightarrow \quad \rho^{\otimes k} \otimes \omega \xrightarrow{(\varepsilon, pm/\lceil n/k \rceil)} \sigma \otimes \omega \quad \Longrightarrow \quad \rho \otimes \omega \xrightarrow{(\varepsilon, pm/n)} \sigma \otimes \omega$$

effectively trading time for space

Our result

$$C_{\varepsilon,p}(\rho,\sigma) \le n/m \quad \Longrightarrow \quad \ddot{C}_{\varepsilon,\, pm\lceil n/k \rceil^{-1}}(\rho,\sigma) \le k, \qquad \forall 1 \le k \le n/m$$

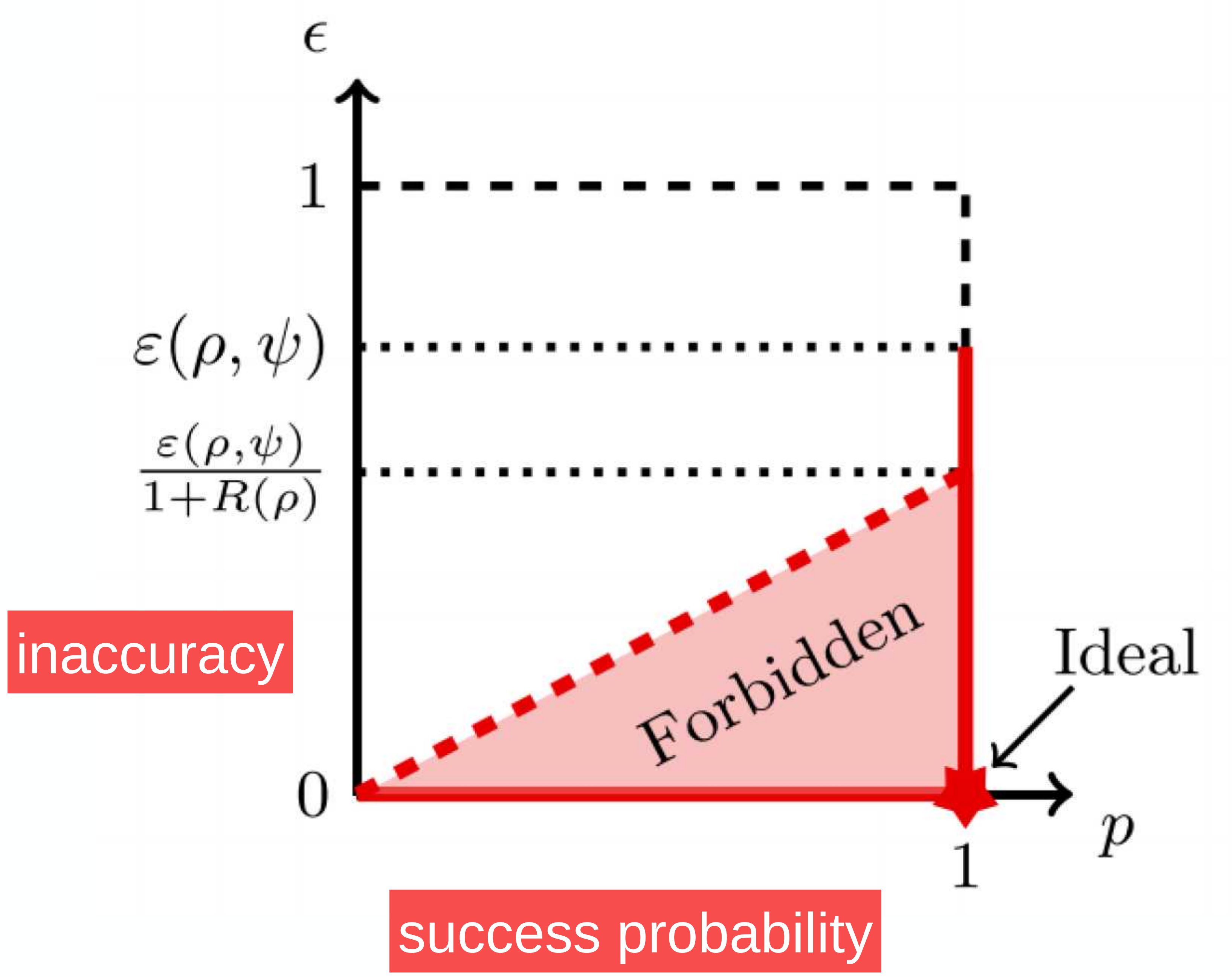
In particular, taking $k = 1$, $\ddot{C}_{\varepsilon,\, pm/n}(\rho,\sigma) = 1$, where m/n can be constant.

Corollary

There exist **one-shot catalytic** magic state distillation protocols that achieve any given target error with constant success probability and **unit overhead** (using only one copy of the source magic state).

Pushing the magic state distillation to its ultimate limit by using catalysts

Remark 1: tradeoff is not trivial without catalysts.



Trading success probability for overhead is not always possible without the use of a catalyst.

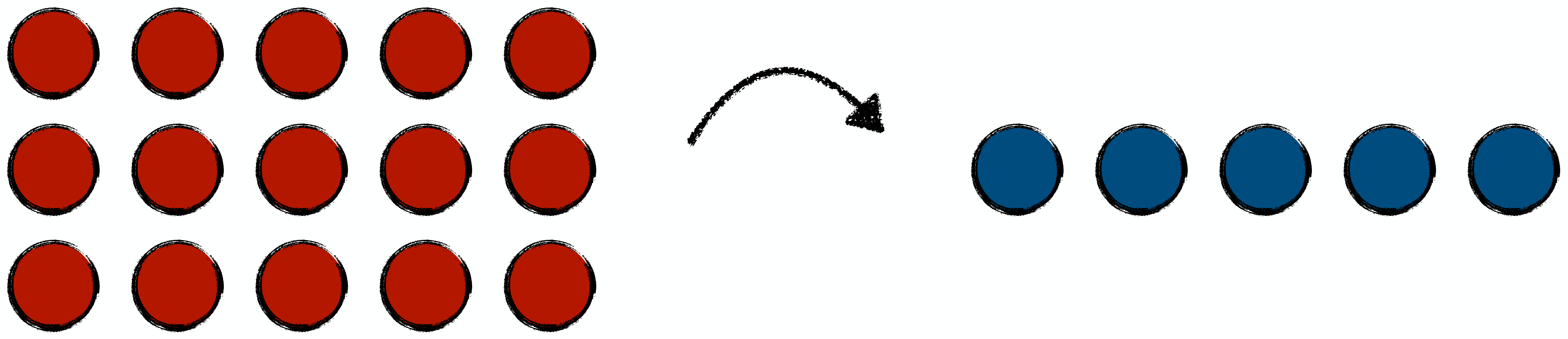
e.g. Entanglement distillation ρ (e.g., an isotropic state) to a Bell state ψ with success probability p' and target error ε', such that (p', ε') is forbidden

However, by the hashing protocol: $\rho^{\otimes mr}$ can be transformed into $\psi^{\otimes m}$ with arbitrarily small target error and success probability 1 for certain r and m, where $1/r$ corresponds to the hashing bound.

A multi-copy transformation does not necessarily guarantee a one-shot transformation by simply compromising the success probability, even when p' is chosen to be close to zero.
Yet, this trade-off can always be achieved with the aid of a catalyst by our result.

Proof idea
from “multi-shot” to “one-shot catalytic”

Proof idea: from multi-shot to one-shot catalytic



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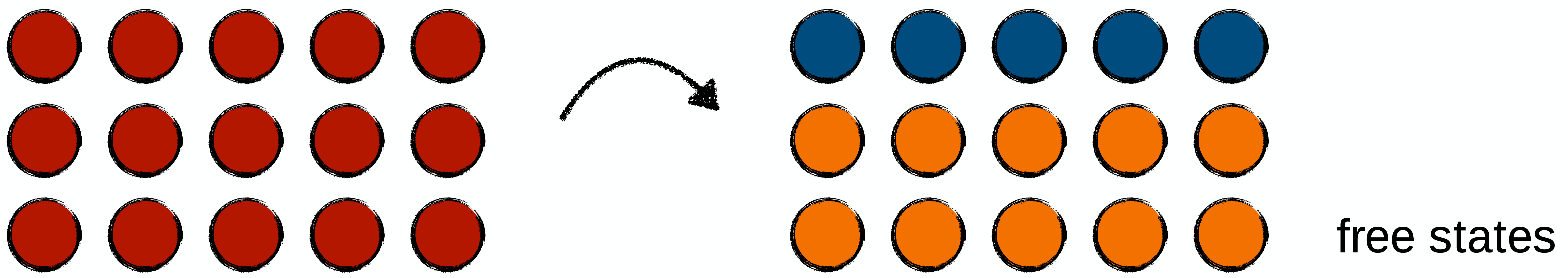

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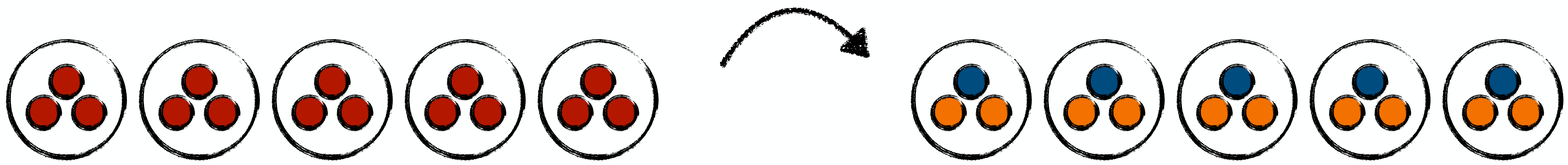

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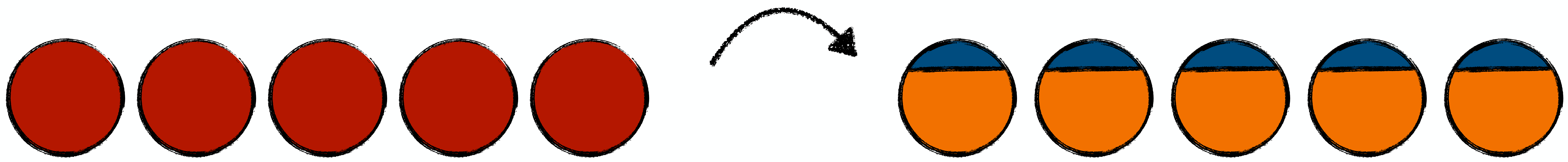

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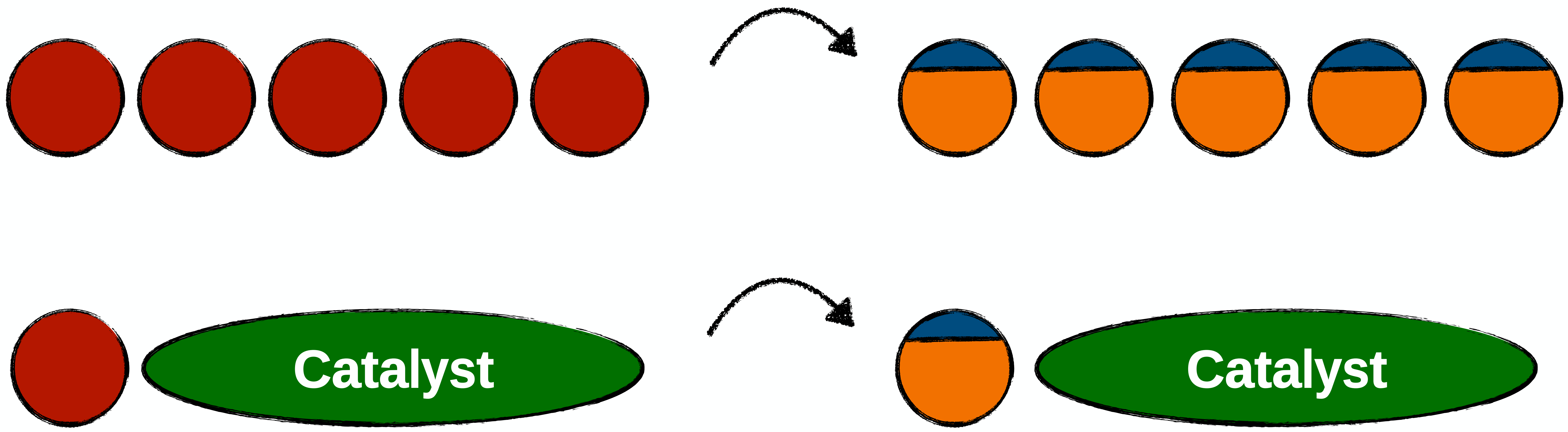

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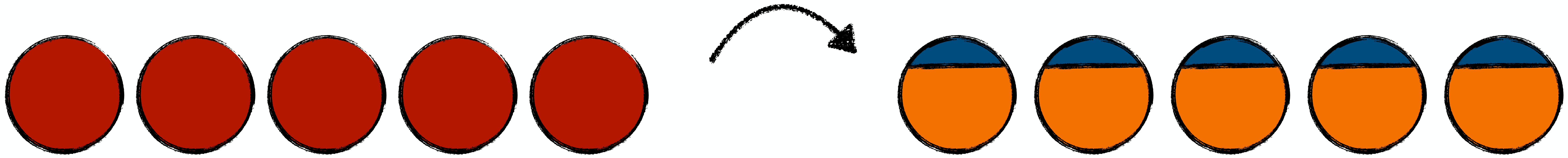
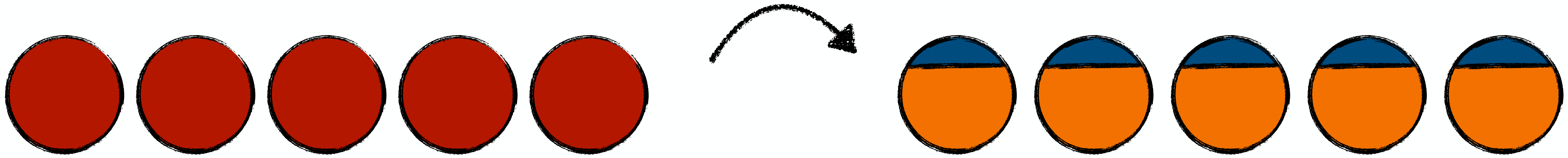


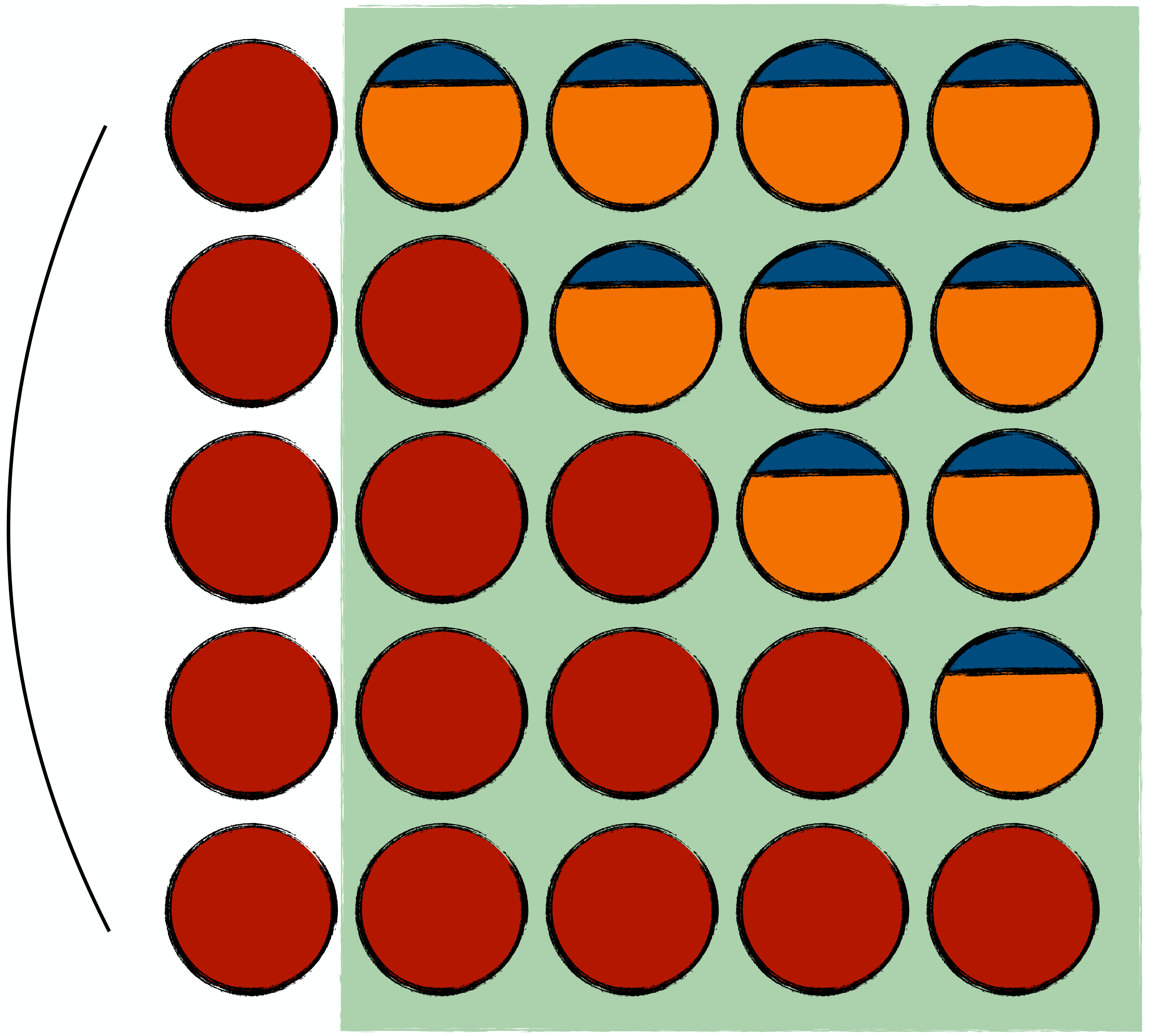
Proof idea: from multi-shot to one-shot catalytic

$\zeta = \rho^{\otimes 3}$



$\hat{\sigma} = \sigma \otimes \pi^{\otimes 2}$

convex mixture



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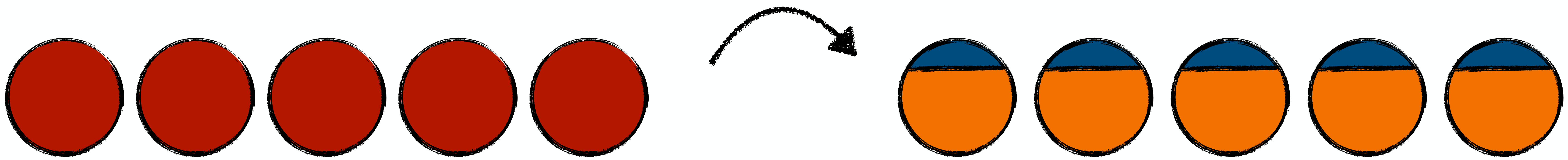

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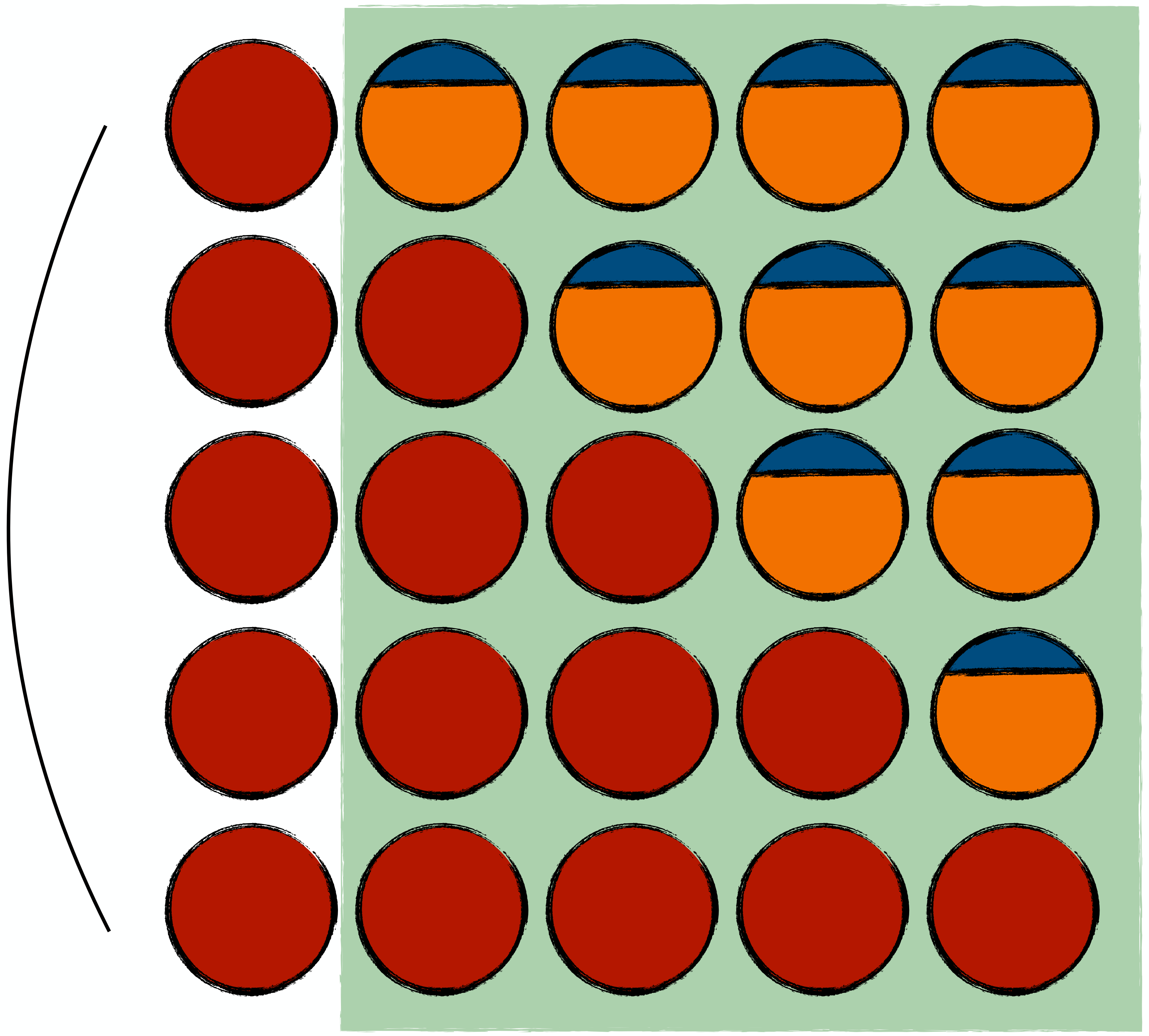

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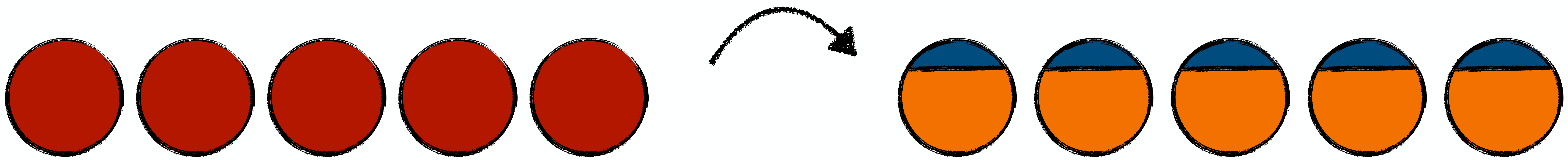

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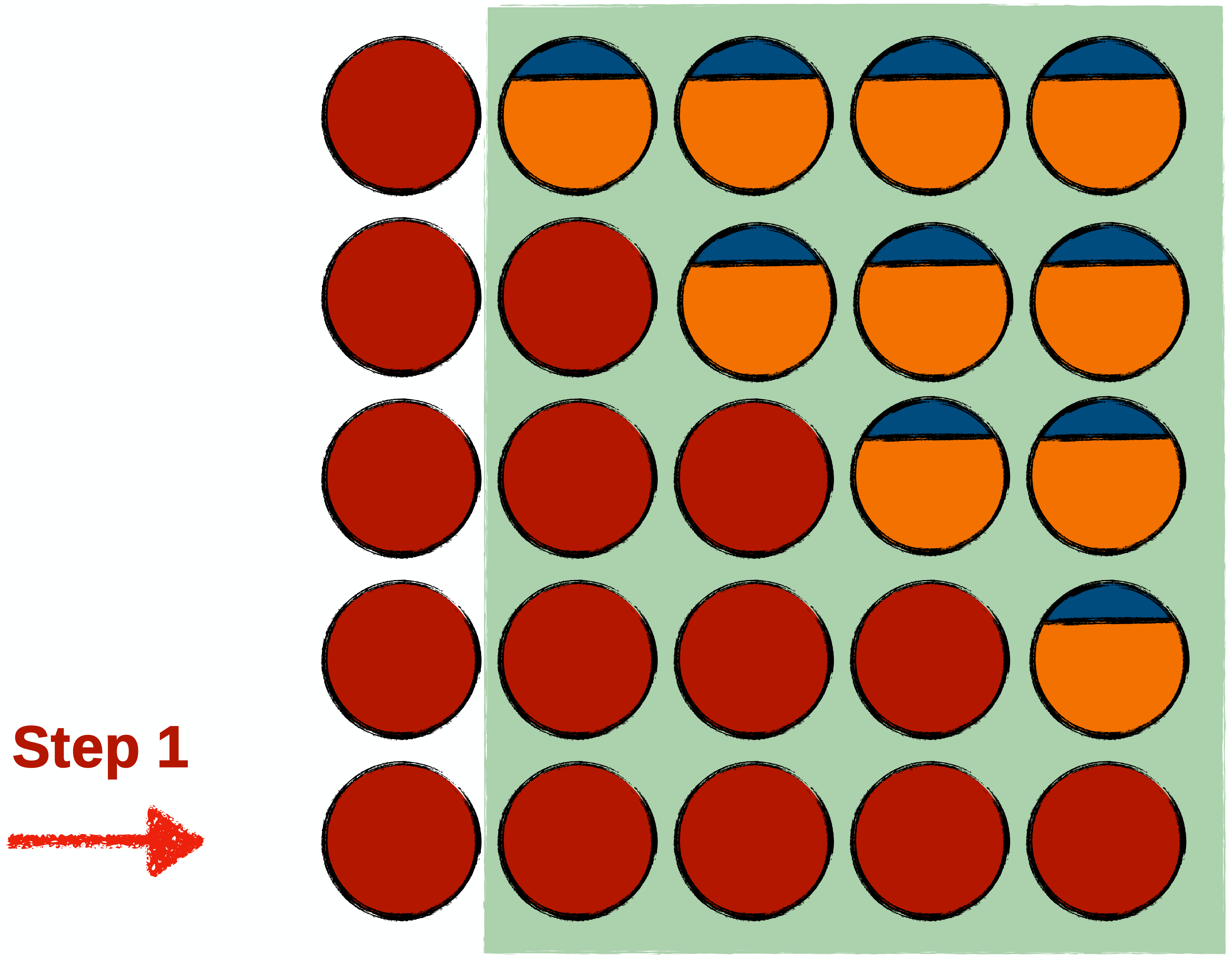

Step 1

$$|1\rangle\langle 1| \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$$

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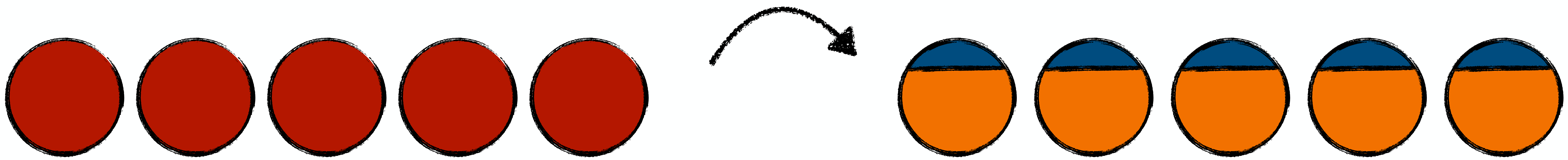
Classically controlled free operation

Proof idea: from multi-shot to one-shot catalytic

$\zeta = \rho^{\otimes 3}$



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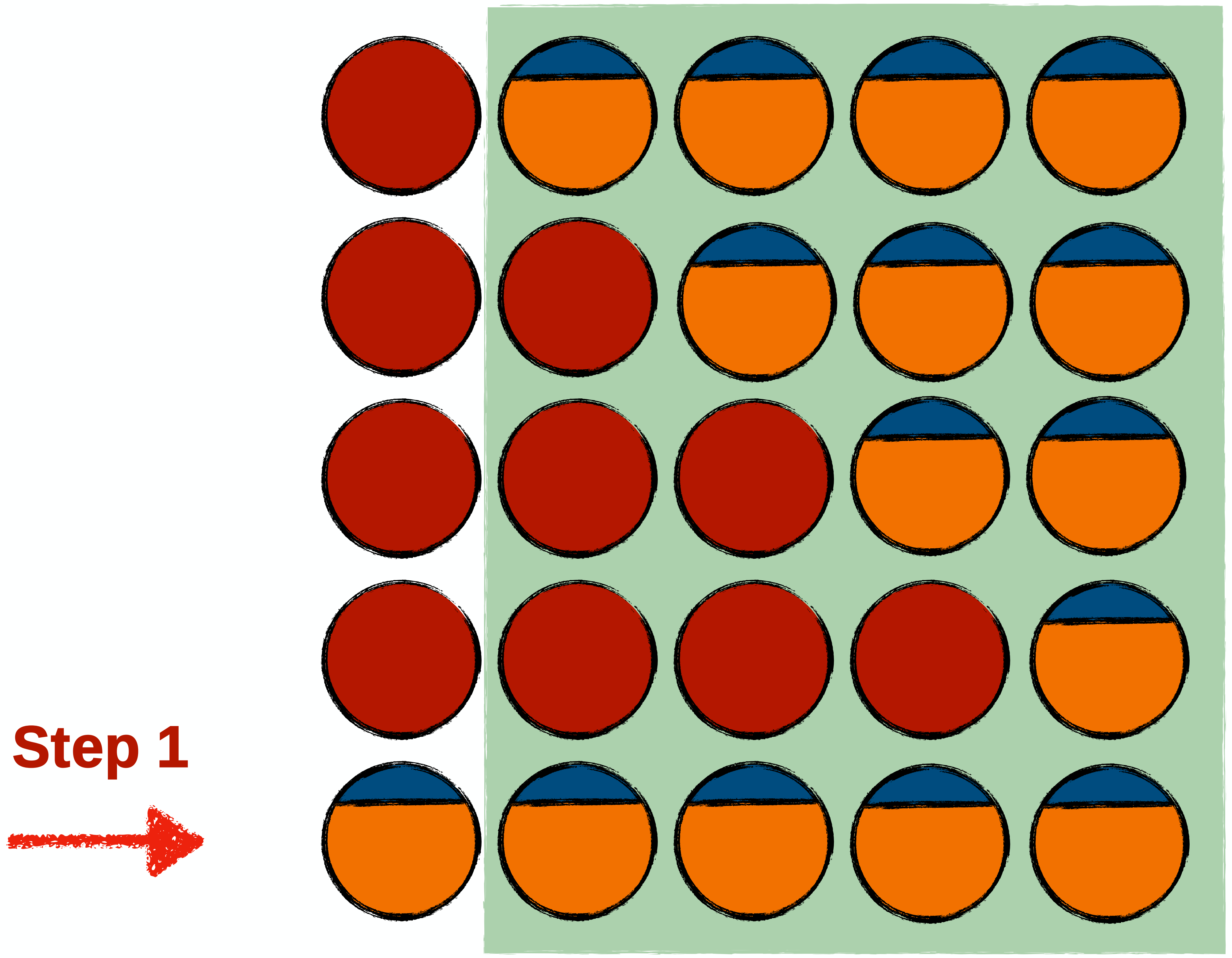

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Proof idea: from multi-shot to one-shot catalytic

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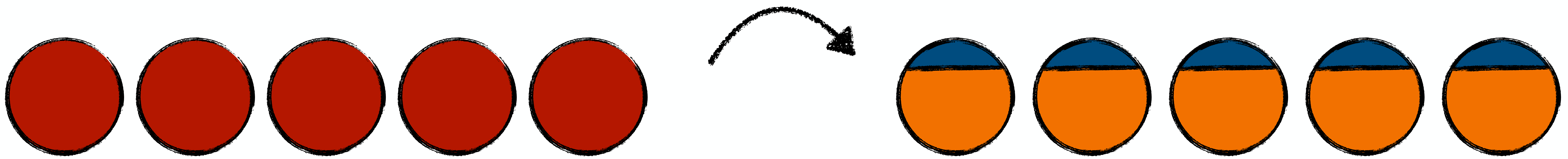

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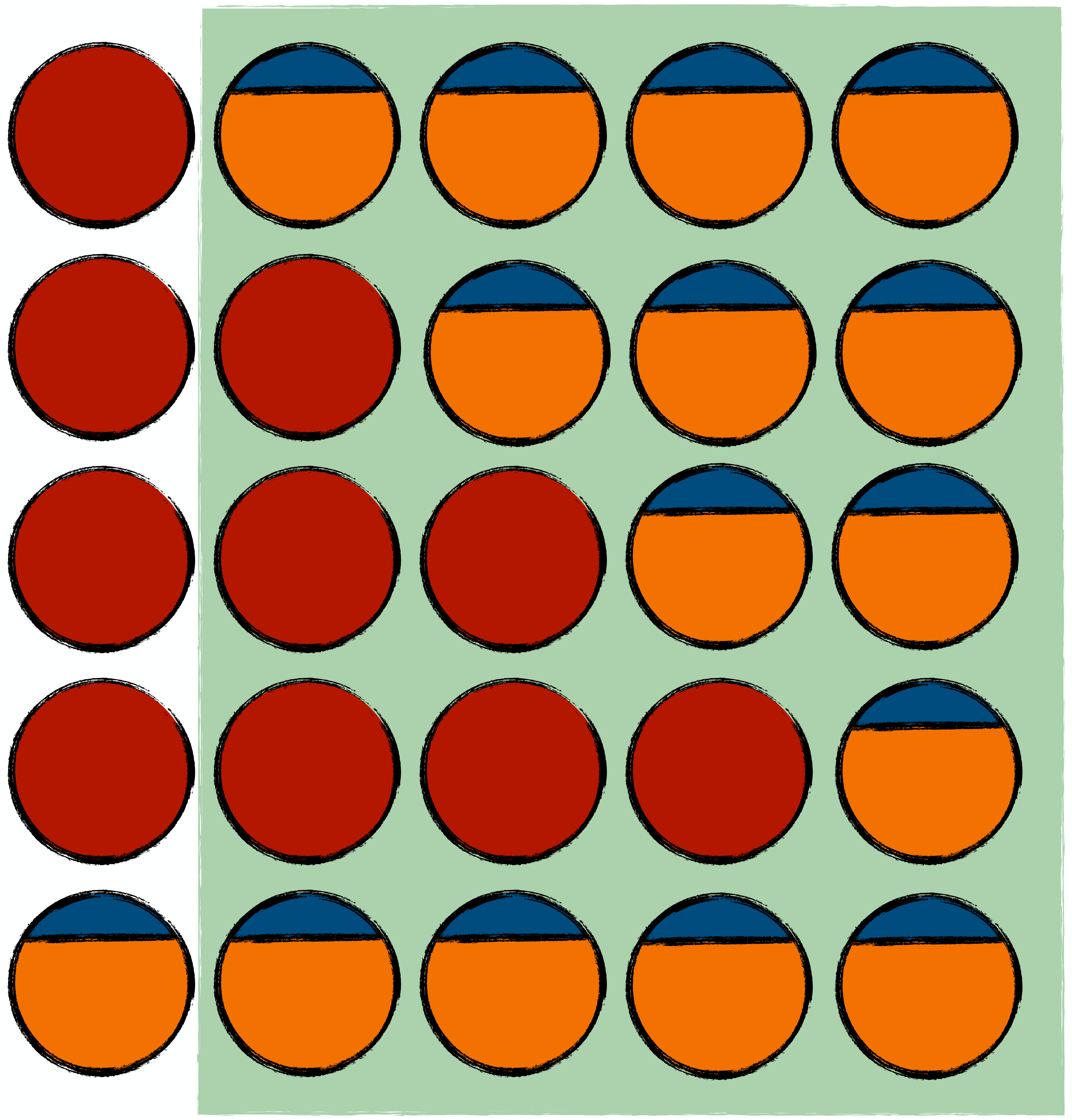

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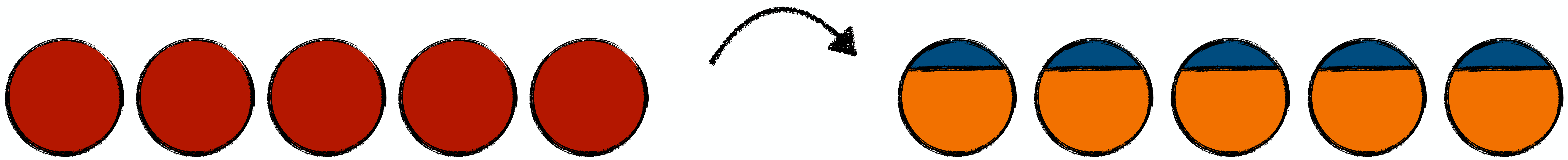
Step 2

Cyclically permuting classical registers

Proof idea: from multi-shot to one-shot catalytic

$\zeta = \rho^{\otimes 3}$



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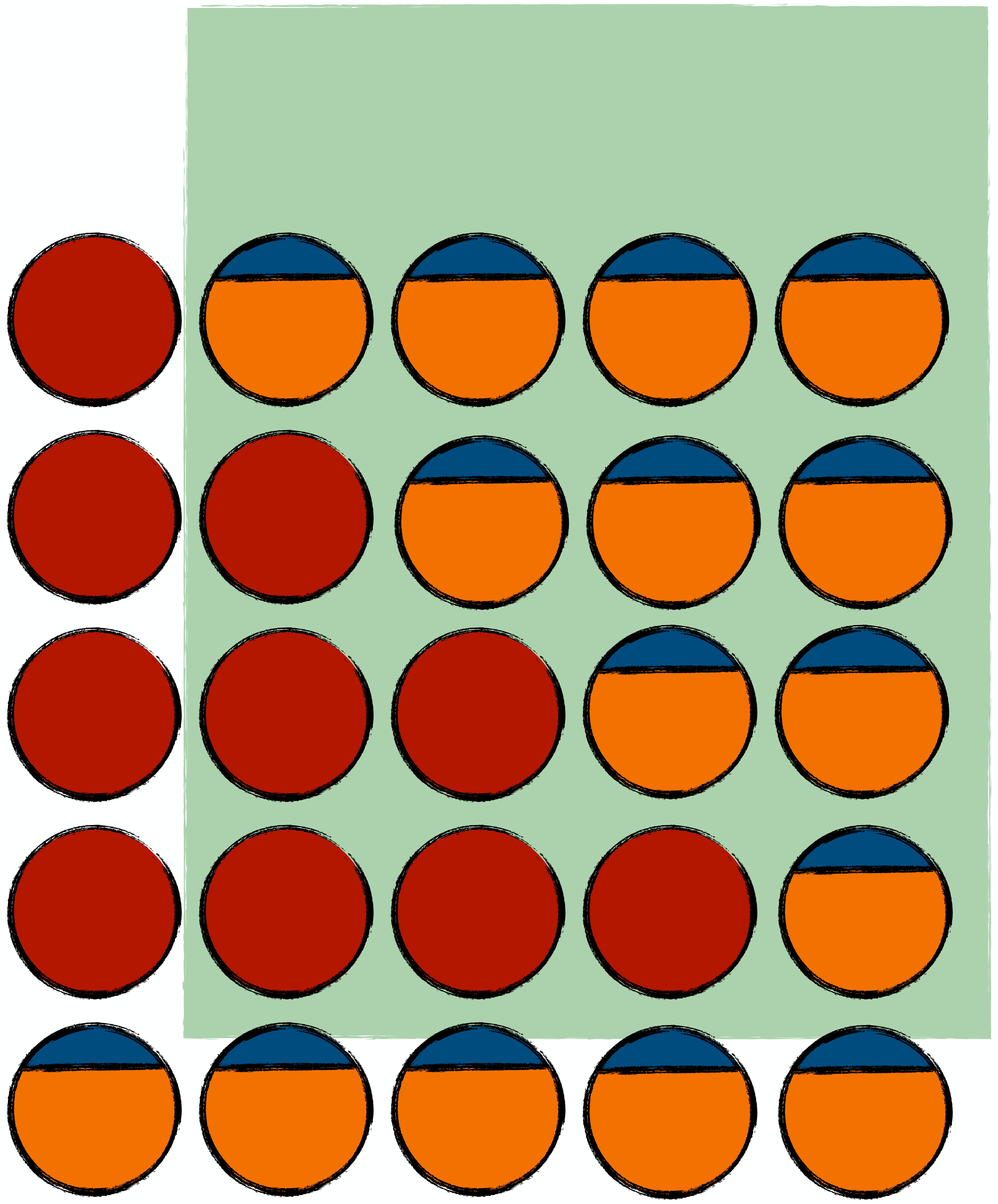

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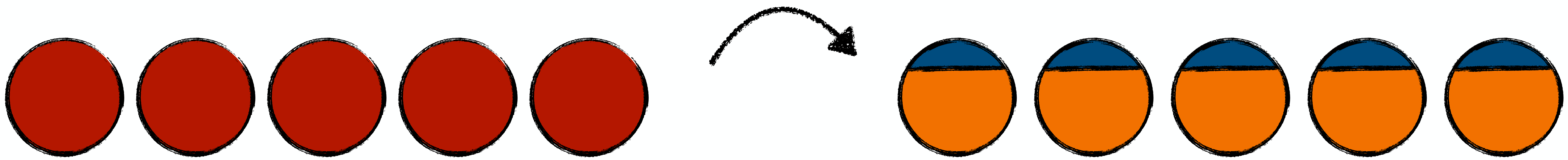
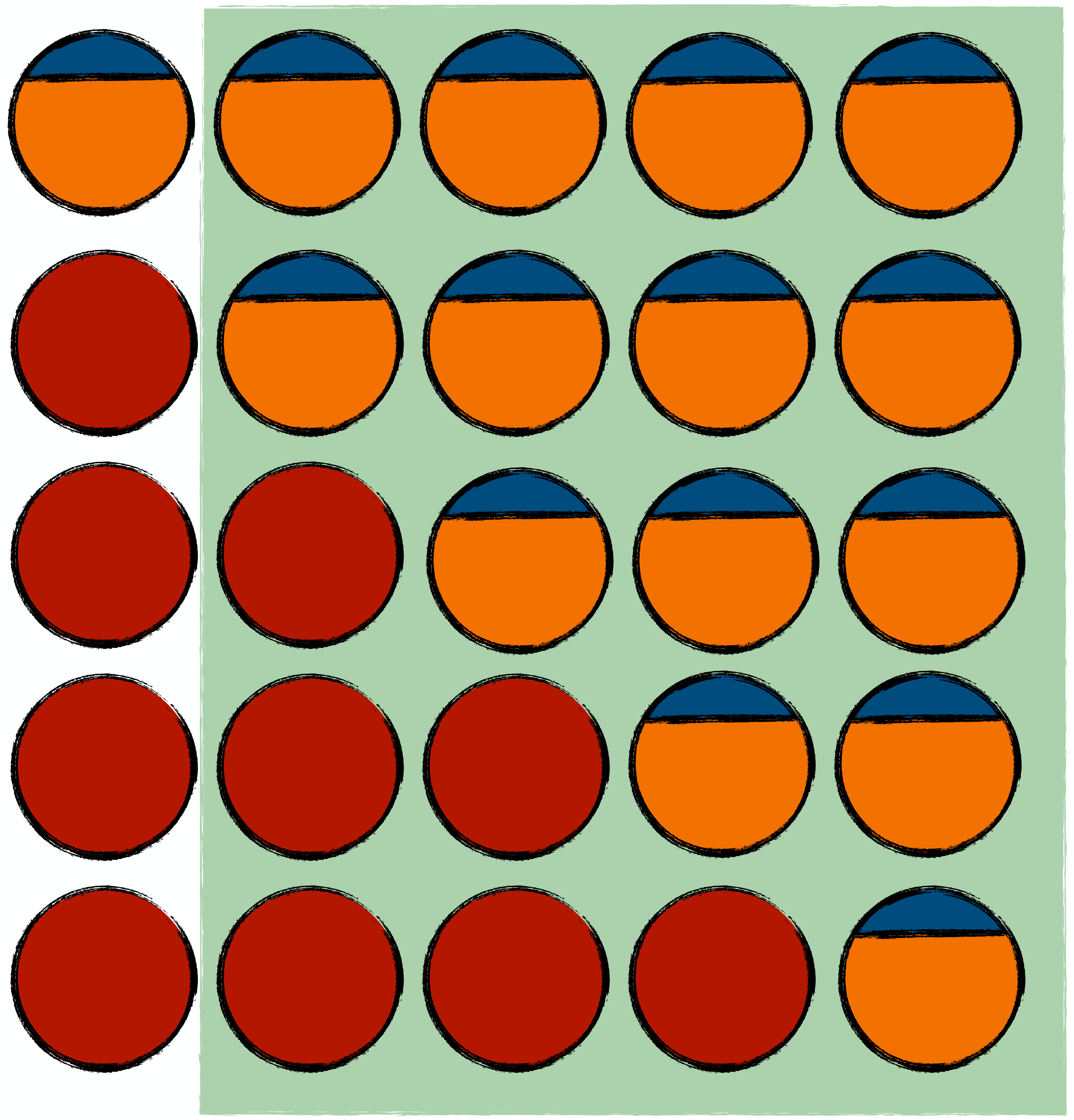
$\zeta = \rho^{\otimes 3}$



$\hat{\sigma} = \sigma \otimes \pi^{\otimes 2}$



$$|1\rangle\langle 1| \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$$

$$|2\rangle\langle 2| \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$$

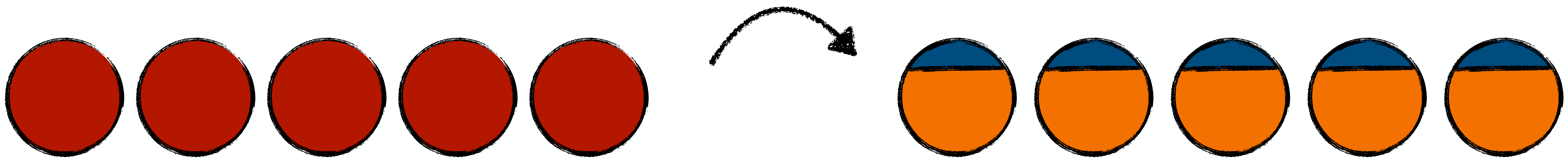
$$|3\rangle\langle 3| \otimes \zeta \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$$

$$|4\rangle\langle 4| \otimes \zeta \otimes \zeta \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma}$$

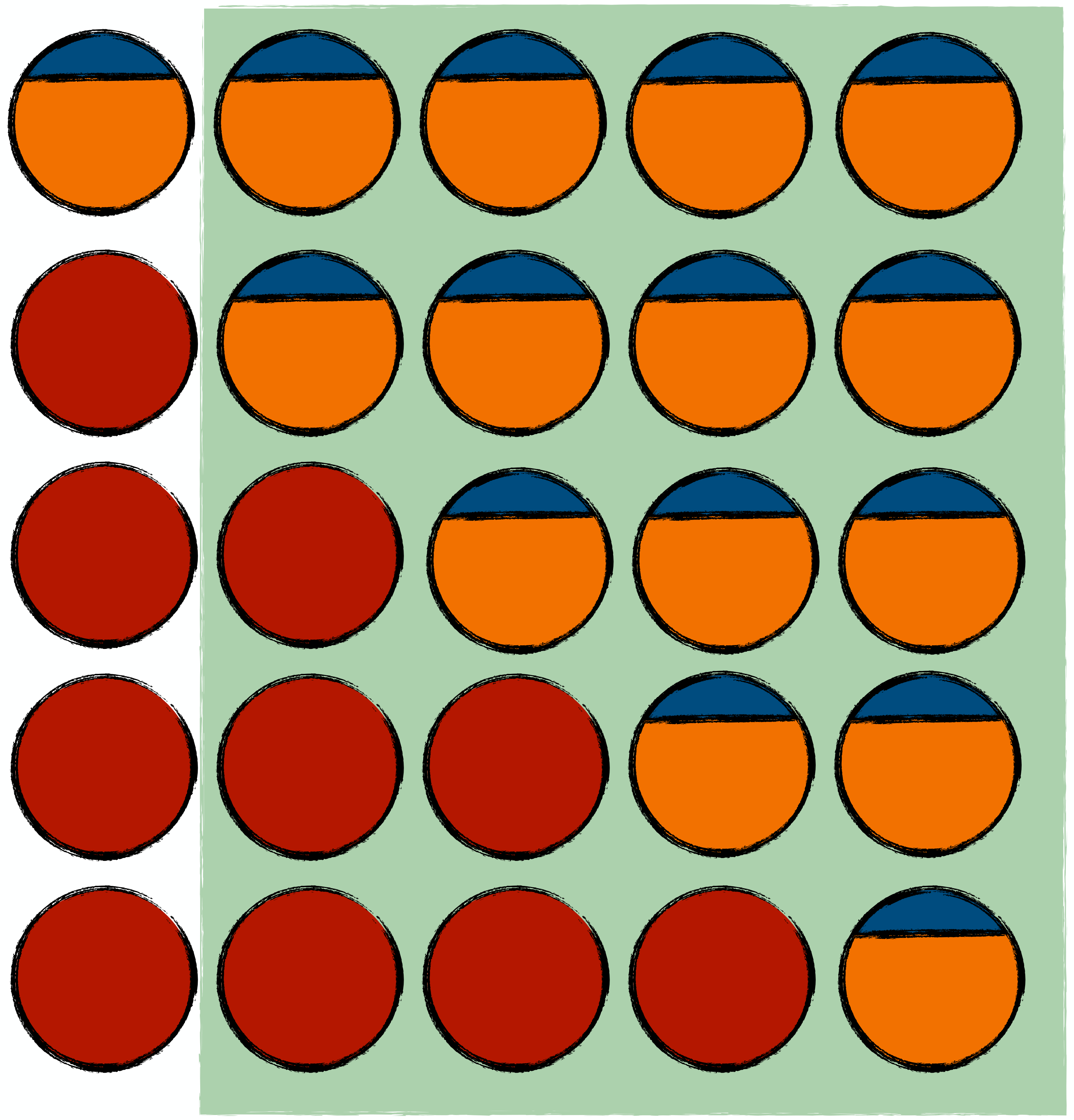
$$|5\rangle\langle 5| \otimes \zeta \otimes \zeta \otimes \zeta \otimes \zeta \otimes \hat{\sigma}$$

Proof idea: from multi-shot to one-shot catalytic

$\zeta = \rho^{\otimes 3}$



$\hat{\sigma} = \sigma \otimes \pi^{\otimes 2}$



$|1\rangle\langle 1| \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|2\rangle\langle 2| \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|3\rangle\langle 3| \otimes \zeta \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|4\rangle\langle 4| \otimes \zeta \otimes \zeta \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma}$

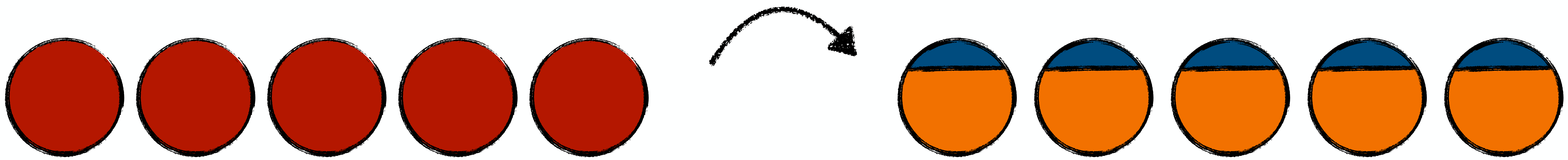
$|5\rangle\langle 5| \otimes \zeta \otimes \zeta \otimes \zeta \otimes \zeta \otimes \hat{\sigma}$

Step 3

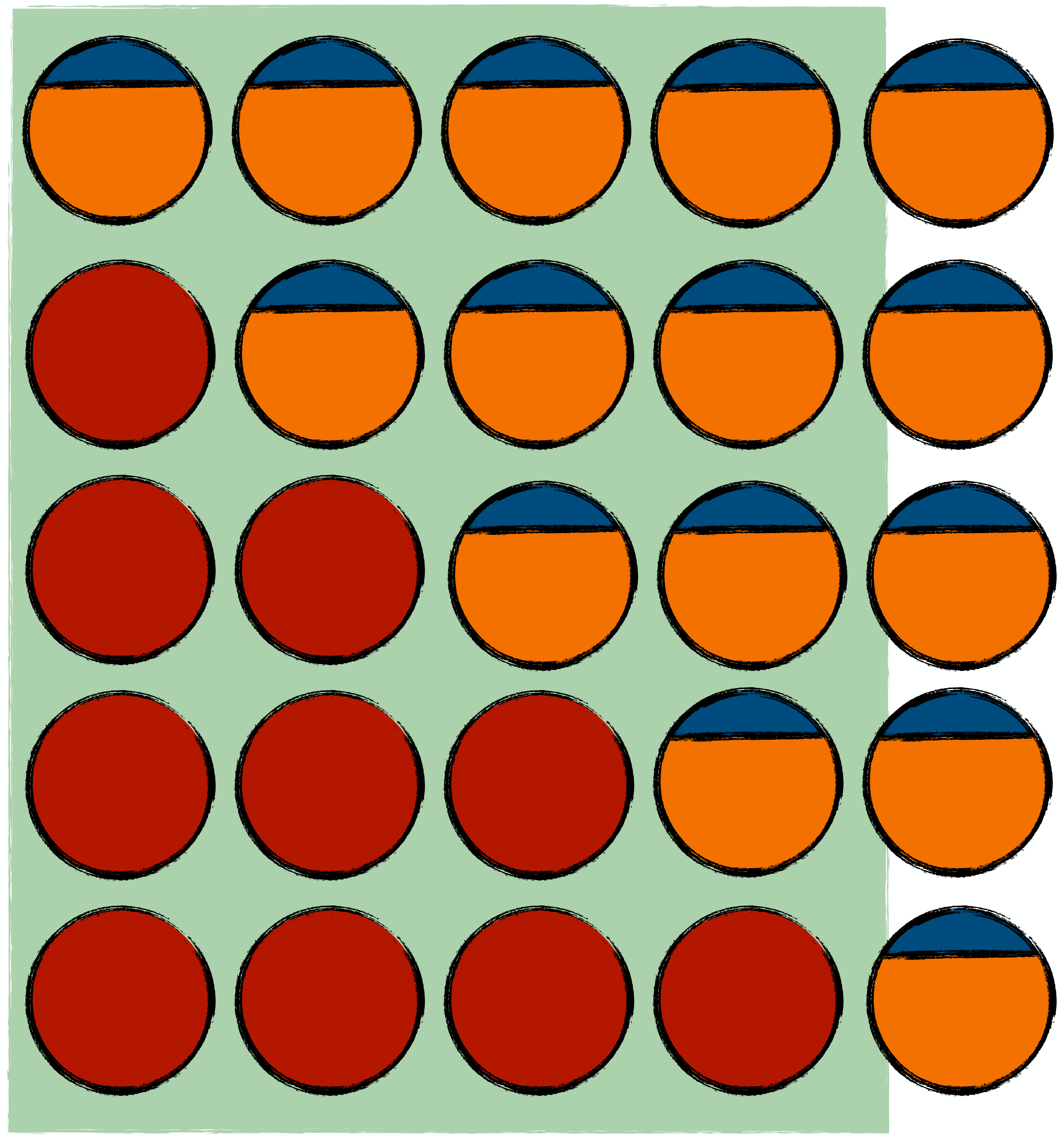
Cyclically permuting quantum registers

Proof idea: from multi-shot to one-shot catalytic

$\zeta = \rho^{\otimes 3}$



$\hat{\sigma} = \sigma \otimes \pi^{\otimes 2}$



$|1\rangle\langle 1| \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|2\rangle\langle 2| \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|3\rangle\langle 3| \otimes \zeta \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|4\rangle\langle 4| \otimes \zeta \otimes \zeta \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma}$

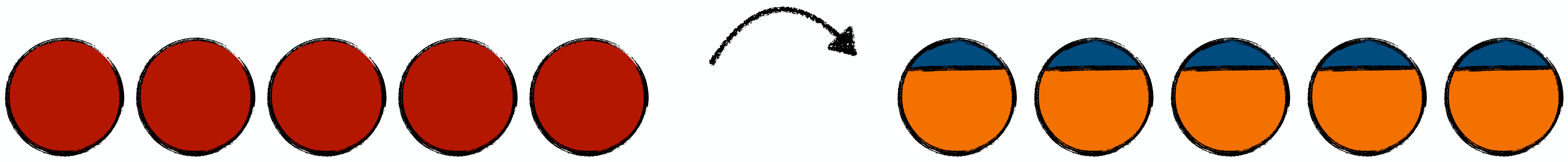
$|5\rangle\langle 5| \otimes \zeta \otimes \zeta \otimes \zeta \otimes \zeta \otimes \hat{\sigma}$

Step 3

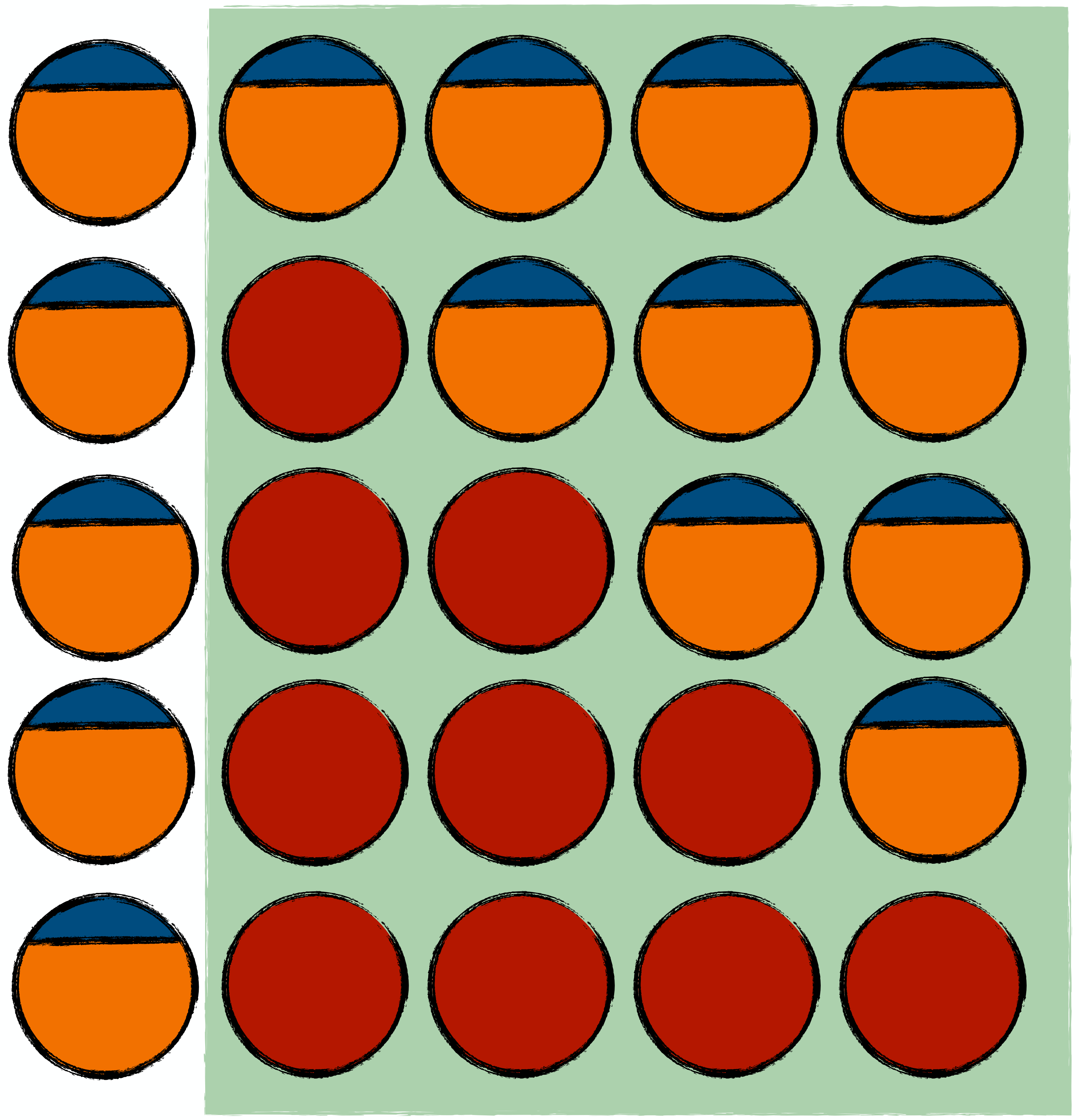
Cyclically permuting quantum registers

Proof idea: from multi-shot to one-shot catalytic

$\zeta = \rho^{\otimes 3}$



$\hat{\sigma} = \sigma \otimes \pi^{\otimes 2}$



$|1\rangle\langle 1| \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|2\rangle\langle 2| \otimes \hat{\sigma} \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma} \otimes \hat{\sigma}$

$|3\rangle\langle 3| \otimes \hat{\sigma} \otimes \zeta \otimes \zeta \otimes \hat{\sigma} \otimes \hat{\sigma}$

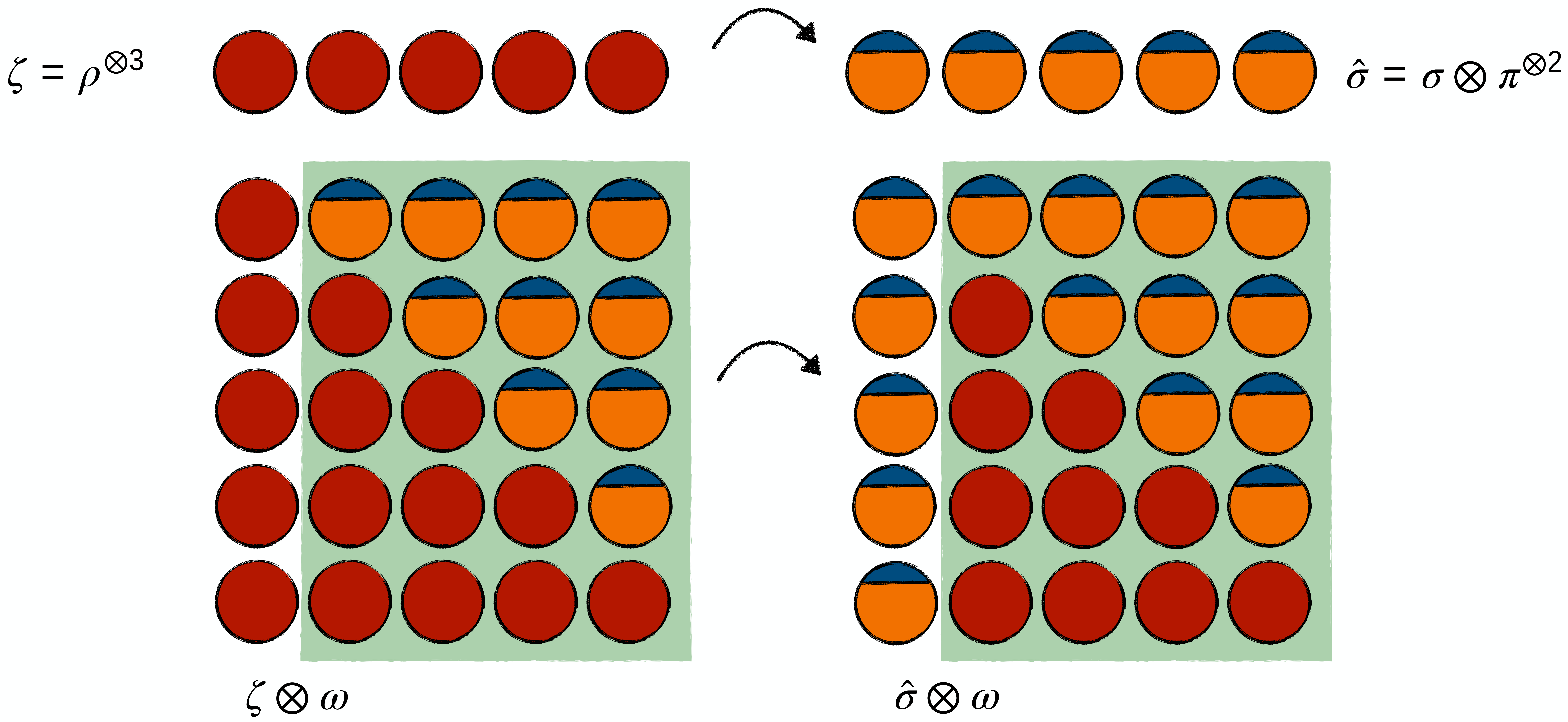
$|4\rangle\langle 4| \otimes \hat{\sigma} \otimes \zeta \otimes \zeta \otimes \zeta \otimes \hat{\sigma}$

$|5\rangle\langle 5| \otimes \hat{\sigma} \otimes \zeta \otimes \zeta \otimes \zeta \otimes \zeta$

Step 3

Cyclically permuting quantum registers

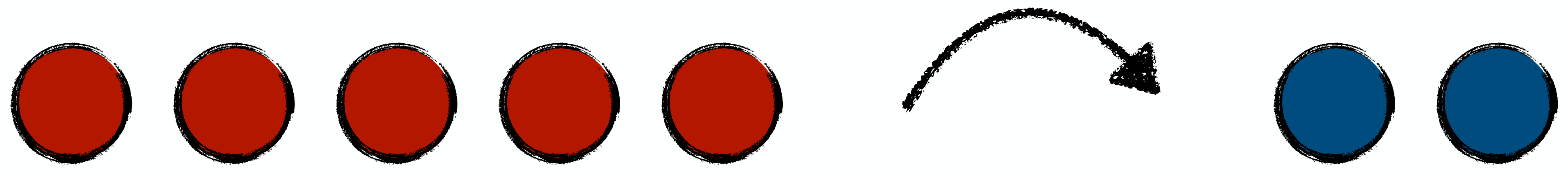
Proof idea: from multi-shot to one-shot catalytic

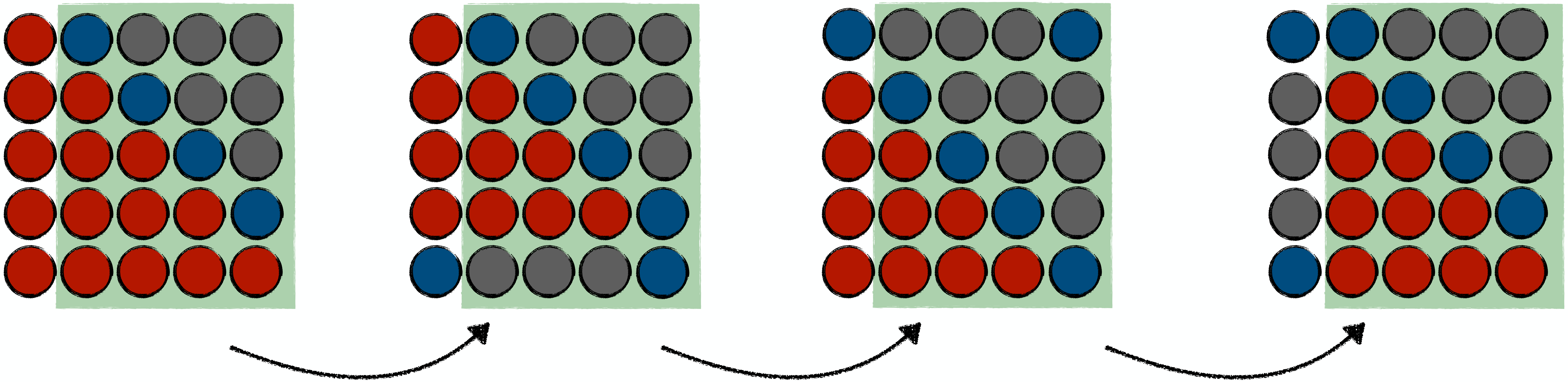


Proof idea

Trading probability for overhead

Proof idea: trading probability for overhead

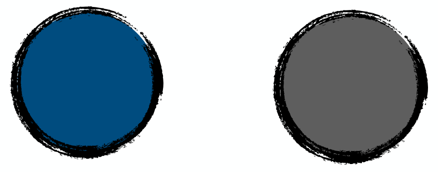
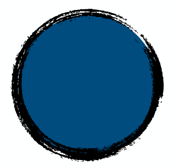




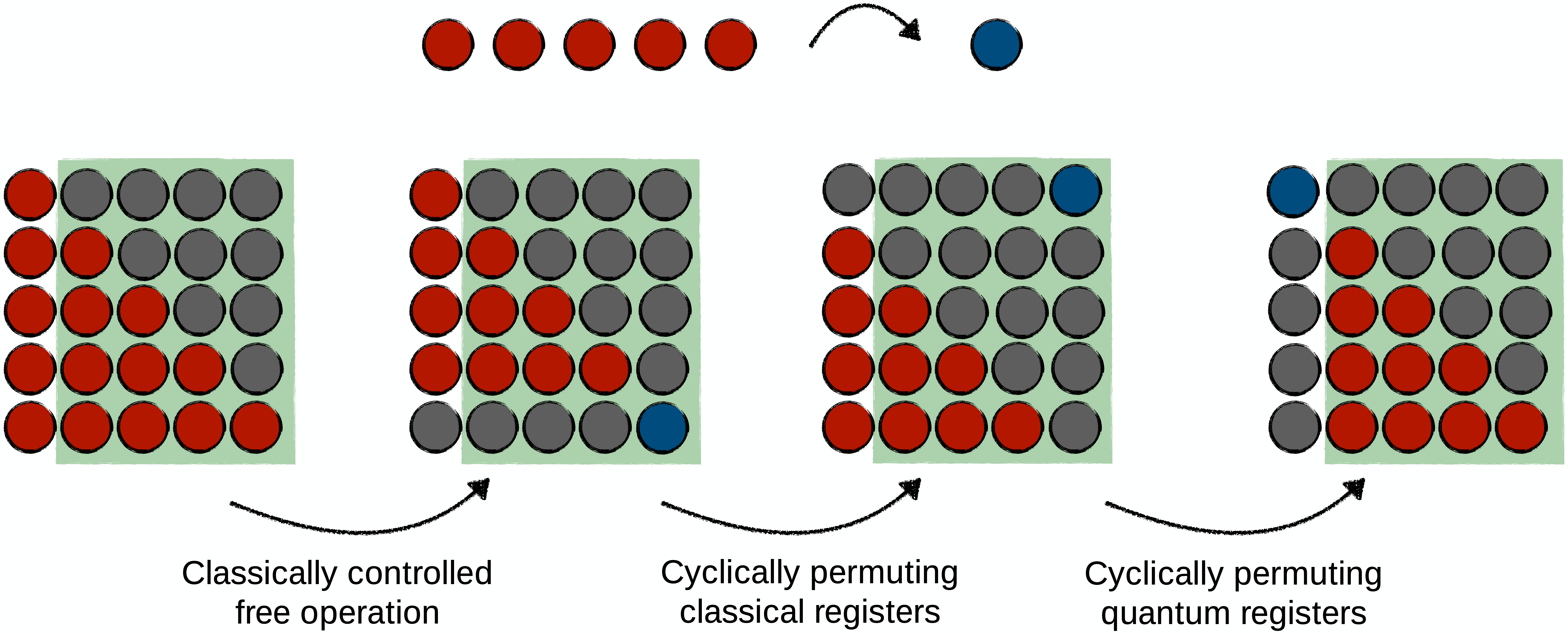
Classically controlled free operation

Cyclically permuting classical registers

Cyclically permuting quantum registers

 Orthogonal,  can be postselected with prob. $n/m = 5/2$

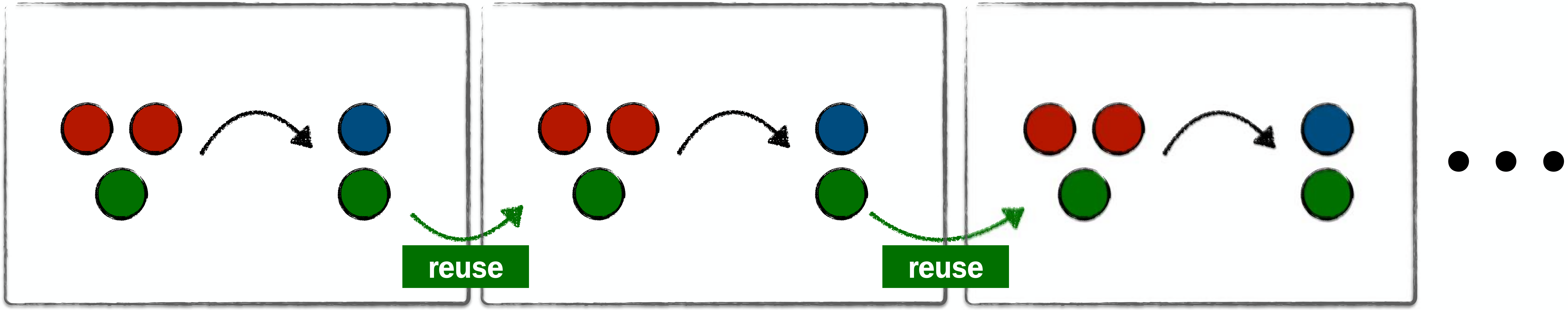
Proof idea: trading probability for overhead



- Catalyst is not unique and we can construct other solutions.
- **When $m = 1$, catalyst can be made independent of the target state**

Remark 2: catalyst reuse

A key advantage of using a catalyst is its recoverability after the transformation, allowing for repeated reuse.

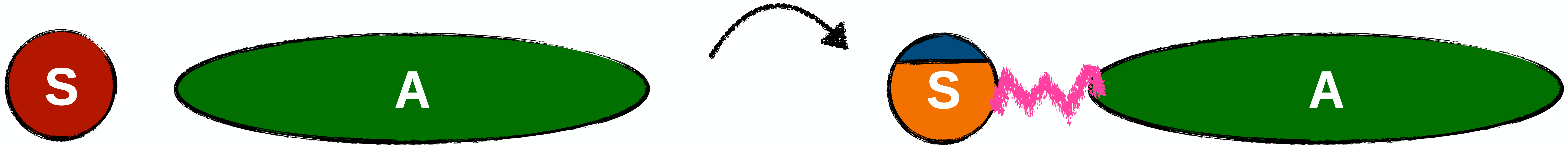


Our result

For a deterministic one-shot catalytic distillation, after $l \geq 1$ repeated uses of the catalyst ω_A, we obtain a joint state $\nu_{S_1 S_2 \cdots S_l A}$ such that

- the catalyst is exactly returned on its marginal $\nu_A = \omega_A$ and
- the target states $\nu_{S_1} = \nu_{S_2} = \cdots = \nu_{S_l}$ with error $\Delta(\nu_{S_i}, \sigma_S) \leq \varepsilon$ for all i.

Remark 3: correlations



Catalyst may exhibit correlations with the remaining systems

Arbitrarily small correlations requires a divergent amount of resources in the catalyst if the resource theory has multiplicative maximum fidelity of resource **[Rubboli-Tomamichel-22]**

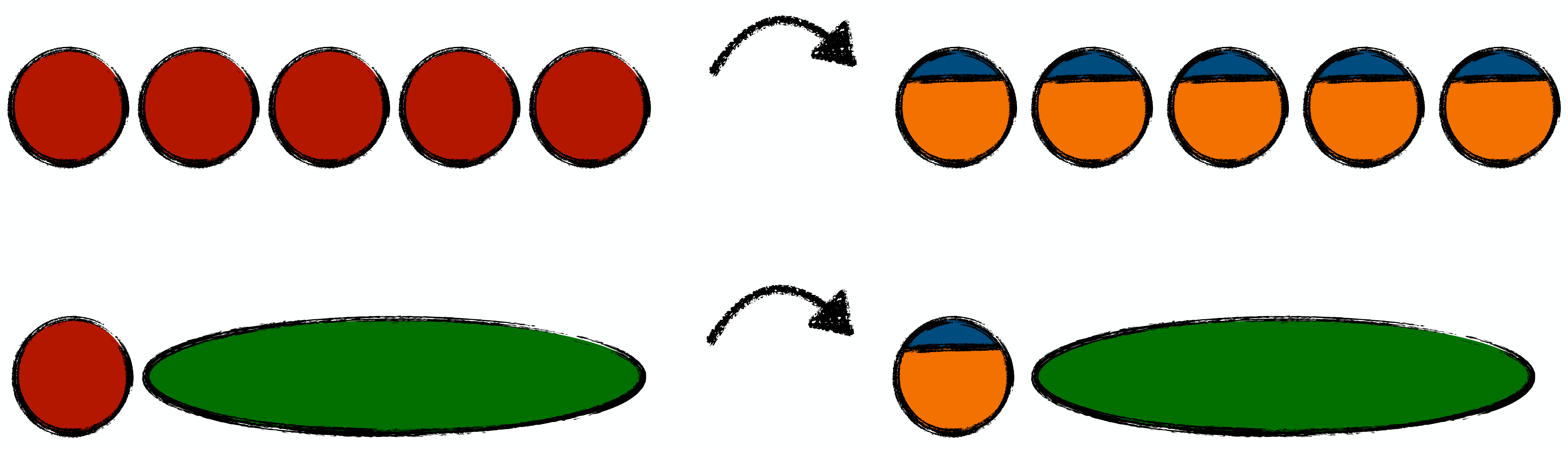
$\hat{F}(\rho_1 \otimes \rho_2) = \hat{F}(\rho_1) \cdot \hat{F}(\rho_2)$, where $\hat{F}(\rho) := \max_{\sigma \in \mathcal{F}} F(\rho, \sigma)$, F state fidelity.

However, it is known that this assumption is not satisfied for magic [Bravyi et al.-19]

Moreover, for **pure target states**, the correlation is under control [Ganardi-Kondra-Streltsov-23].

$$\Delta(\nu_S, |\varphi\rangle\langle\varphi|_S) \leq \varepsilon \text{ implies } \Delta(\nu_{SA}, |\varphi\rangle\langle\varphi|_S \otimes \nu_A) \leq \varepsilon + 3\sqrt{\varepsilon}.$$

Remark 4: catalyst size and dependence



[Duan-Feng-Li-Ying-05-PRA]
[Char-Chakraborty-Bhar-Chattopadhyay-Sarkar-23-PRA]
[Datta-Ganardi-Kondra-Streltsov-23-PRL]
[Kondra-Datta-Streltsov-21-PRL]
[Lipka Bartosik-Skrzypczyk-21-PRL]
[Shiraishi-Sagawa-21-PRL]
…

- The catalysts used have a size comparable to the system in multi-shot protocols, necessitating the coherent manipulation of large quantum systems.
- The catalyst also depends on the source and target states.

- The technical challenge of manipulating large systems remains (for now) ☹
- Towards a more practical setting (one-shot) ☺
- New perspective to study overhead, new possibility to find better catalysts ☺

Summary

Contributions:

- Confirm the effectiveness of catalytic methods in enhancing distillation overhead
- Establish a general and rigorous ordering of resource overheads for different settings
- Catalysts with reusability guarantees
- **Trading success probability for reduced overhead (spacetime tradeoff)**
- **Pushing the magic state distillation to its ultimate limit by using catalysts (unit overhead)**
- Extend the catalytic technique to the channel setting

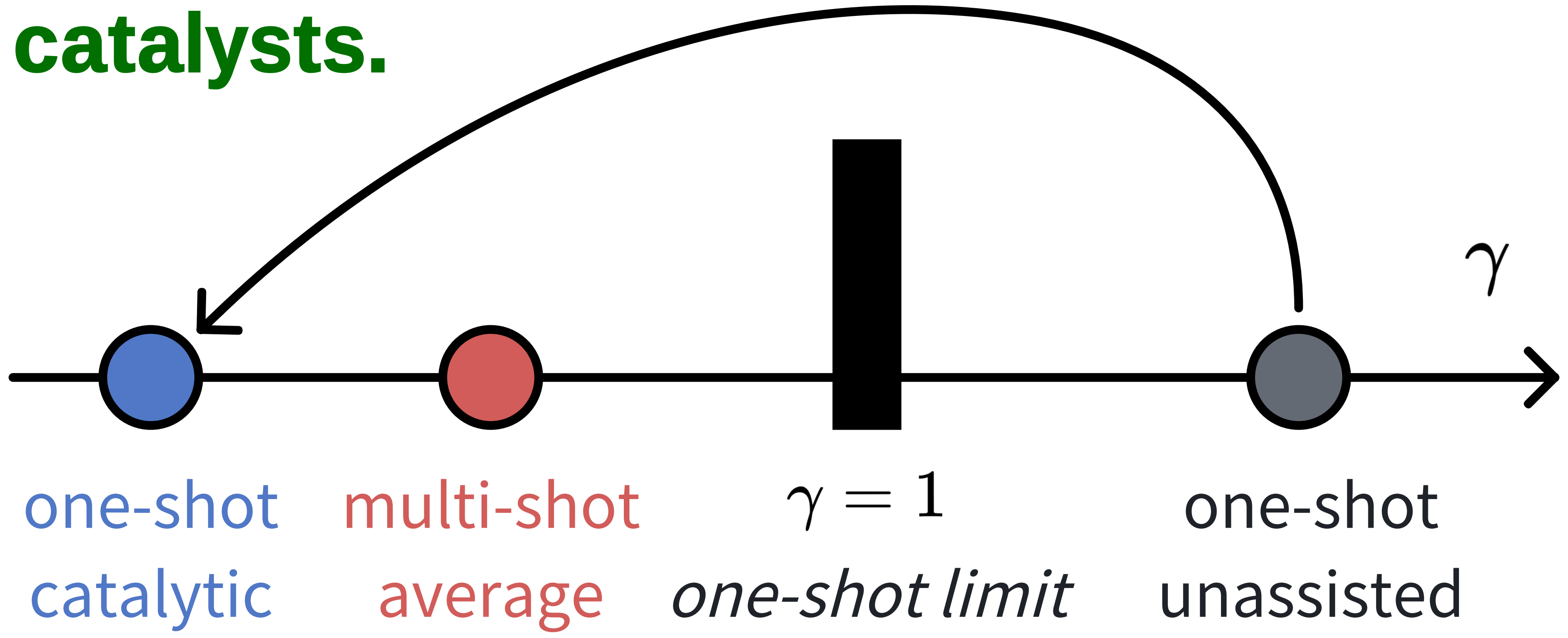
  understand channel mutual information in the one-shot catalytic setting

The story is not completely finished.

But it opens the possibility to design smaller and more efficient catalysts.

Future work:

- Better catalyst with smaller size? State-independent?
- More systematic channel theory?



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