Dynamic quantum circuit compilation and circuit remapping

2310.11021 & PRApplied 22, 024029

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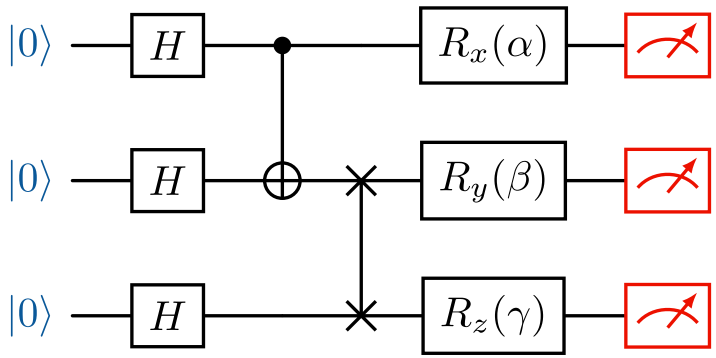
THE UNIVERSITY OF HONG KONG

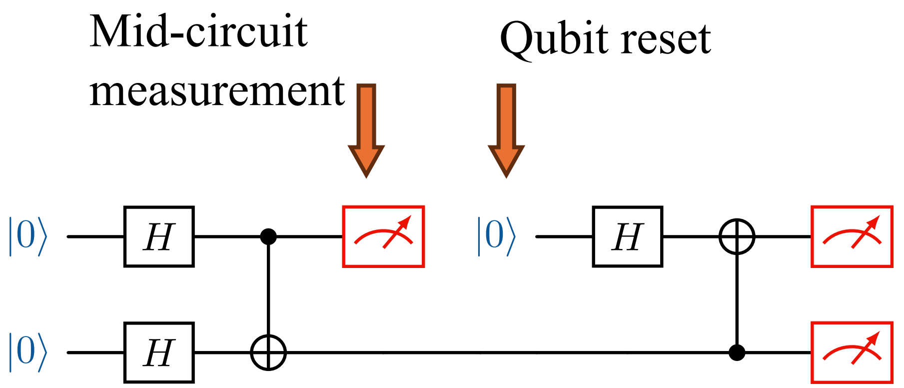
What is Dynamic Quantum Circuit

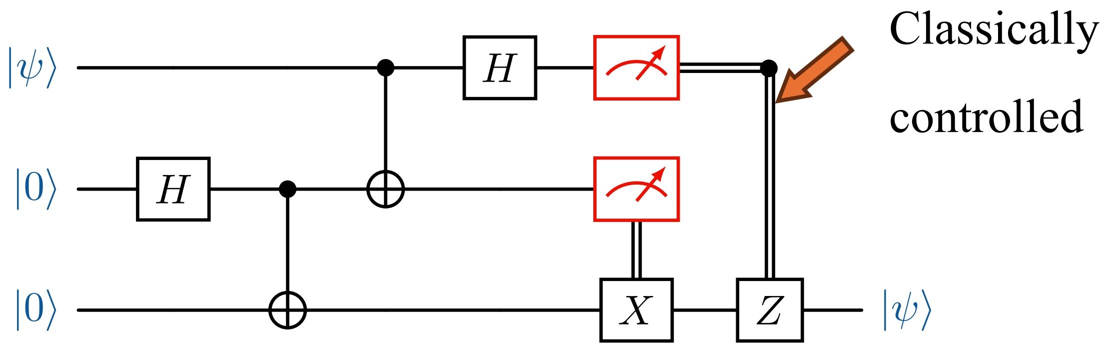
Static Quantum Circuit



- Start with state initialization
- Measurement at the end

Dynamic Quantum Circuit





Hardware Support for Dynamic Quantum Circuit

Exploiting Dynamic Quantum Circuits in a Quantum Algorithm with Superconducting Qubits

A. D. Córcoles, Maika Takita, Ken Inoue, Scott Lekuch, Zlatko K. Minev, Jerry M. Chow, and Jay M. Gambetta
Phys. Rev. Lett. **127**, 100501 – Published 31 August 2021

IBM, 2021

Demonstration of the trapped-ion quantum CCD computer architecture

J. M. Pino, J. M. Dreiling, C. Figgatt, J. P. Gaebler, S. A. Moses, M. S. Allman, C. H. Baldwin, M. Foss-Feig, D. Hayes, K. Mayer, C. Ryan-Anderson & B. Neyenhuis

Nature **592**, 209–213 (2021) | Cite this article

Honeywell, 2021

Suppressing quantum errors by scaling a surface code logical qubit

Google Quantum AI

Superconducting Circuit

Nature **614**, 676–681 (2023) | Cite this article

Google, 2023

Midcircuit Qubit Measurement and Rearrangement in a ^{171}Yb Atomic Array

M. A. Norcia *et al.*
Phys. Rev. X **13**, 041034 (2023) – Published 22 November 2023

Atom Computing, 2023

Beating the break-even point with a discrete-variable-encoded logical qubit

Zhongchu Ni, Sai Li, Xiaowei Deng, Yanyan Cai, Libo Zhang, Weiting Wang, Zhen-Biao Yang, Haifeng Yu, Fei Yan, Song Liu, Chang-Ling Zou, Luyan Sun, Shi-Biao Zheng, Yuan Xu & Dapeng Yu

Nature **616**, 56–60 (2023) | Cite this article

SUSTech, 2023

Two benefits

Scalability

- Existing quantum computers are limited by the **number of qubits** available for computation. We need to **make efficient use of qubits** when designing and executing quantum algorithms.

Dynamic circuit can help to reduce the number of qubit use

Fidelity

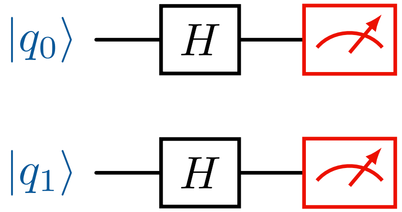
- Current generation of quantum computers are extremely prone to noise. **Circuit optimization** and **error correction** needed !

Dynamic circuit can help to improve the circuit fidelity

What is dynamic circuit compilation

(yield same sampling results)

Rewires static quantum circuits into **<u>equivalent</u>** dynamic circuits using **fewer** qubits through **<u>qubit-reuse</u>** strategies.



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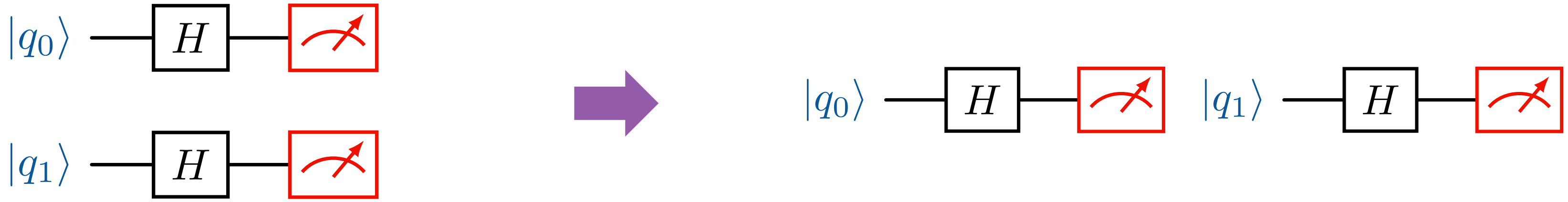

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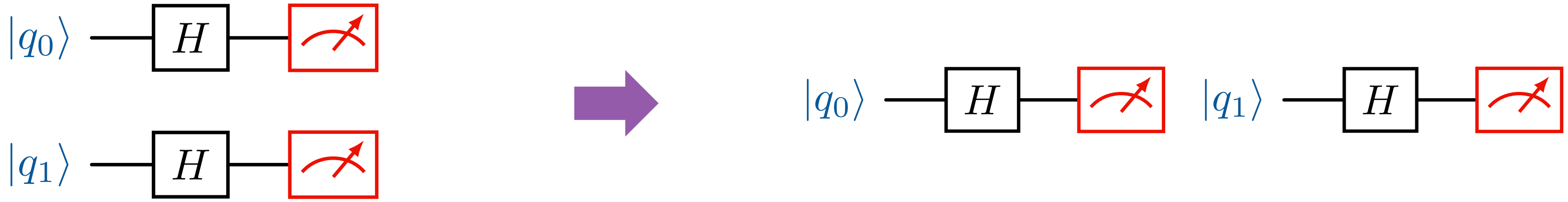

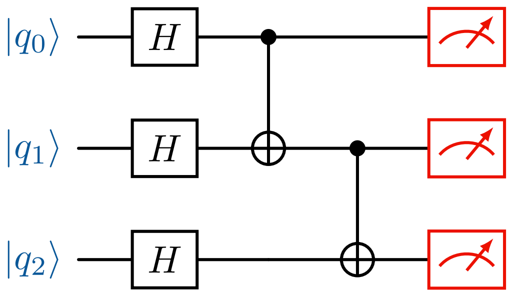

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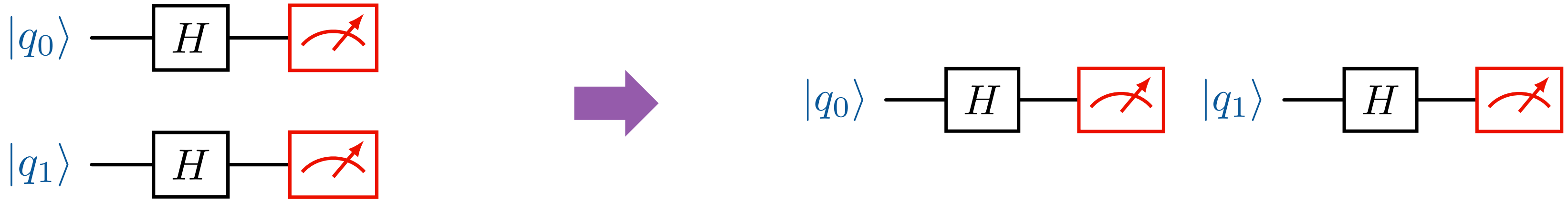

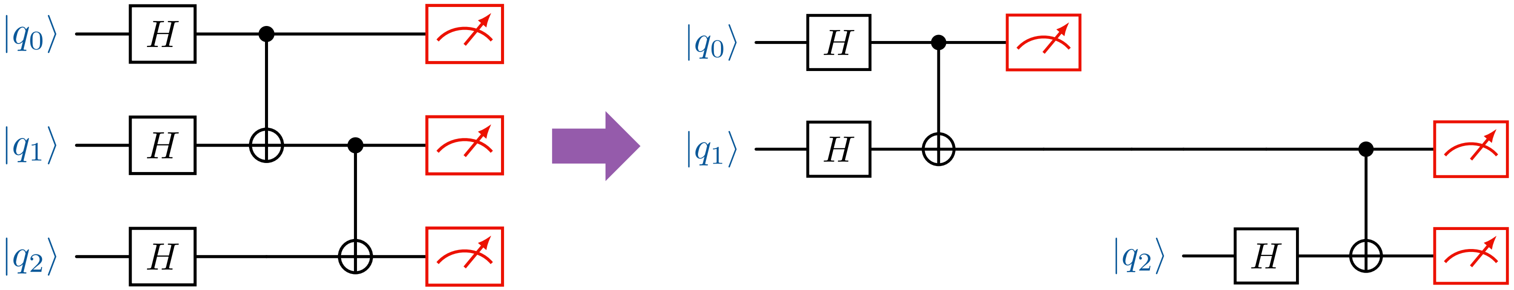

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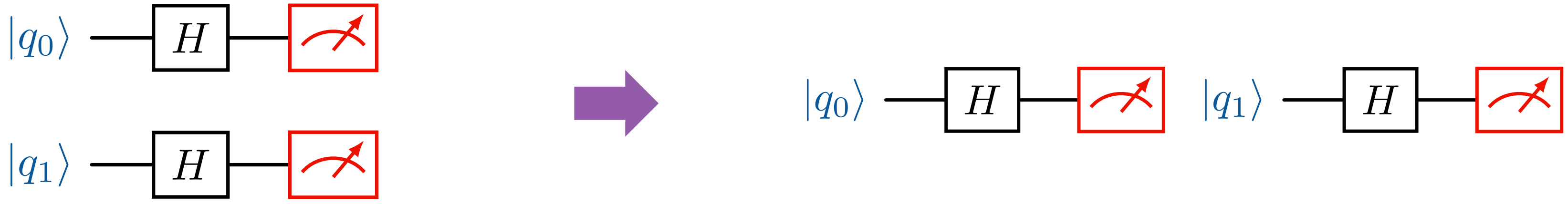

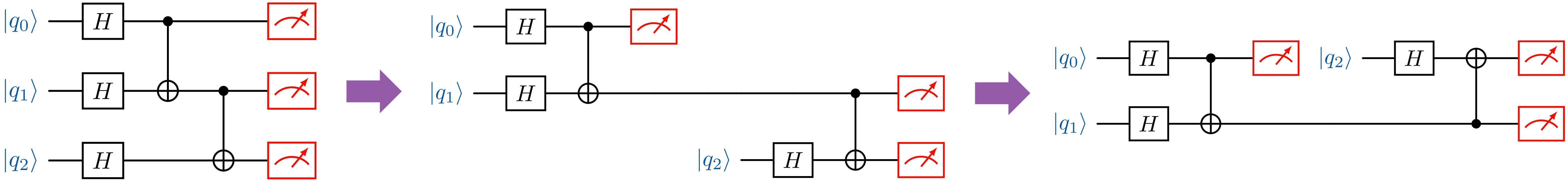

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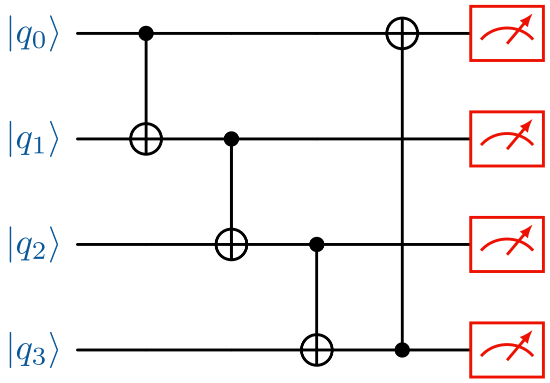

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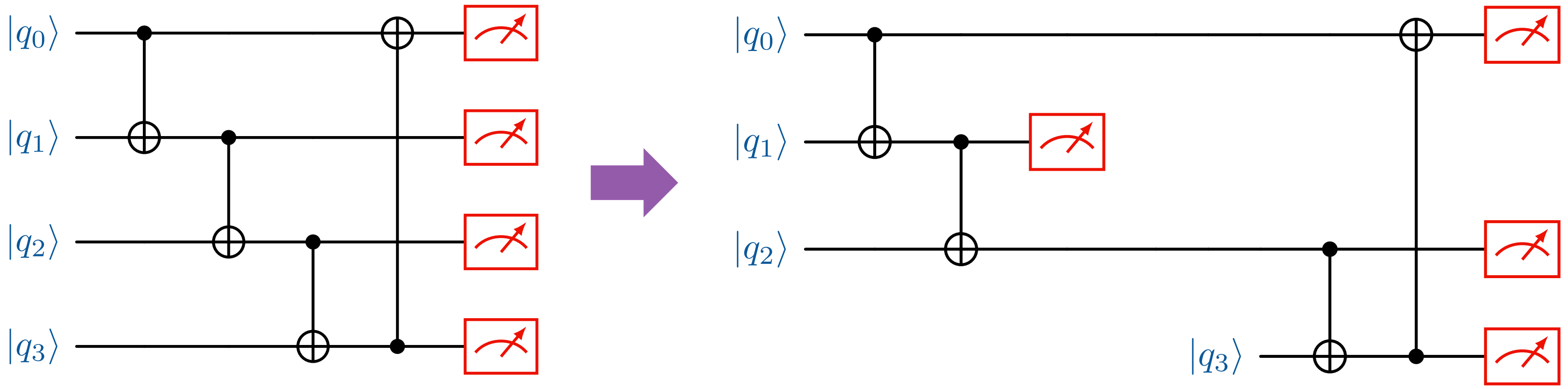

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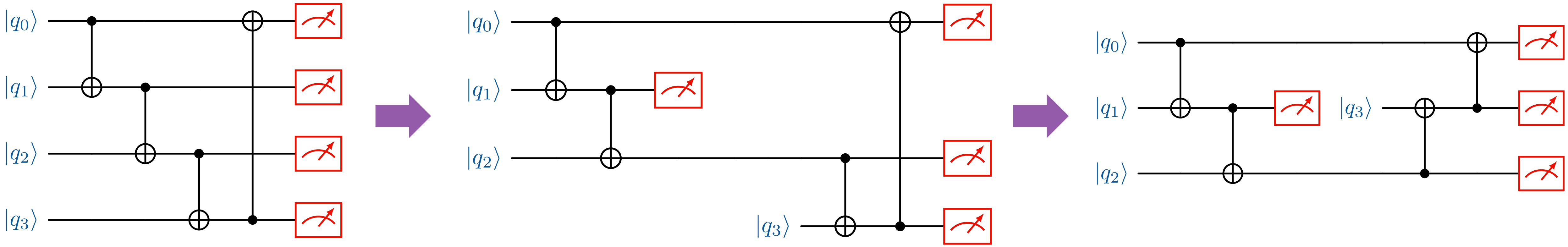

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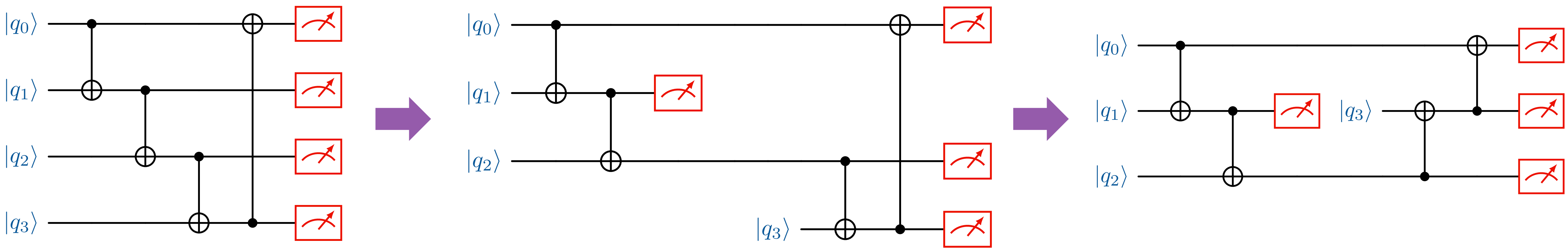
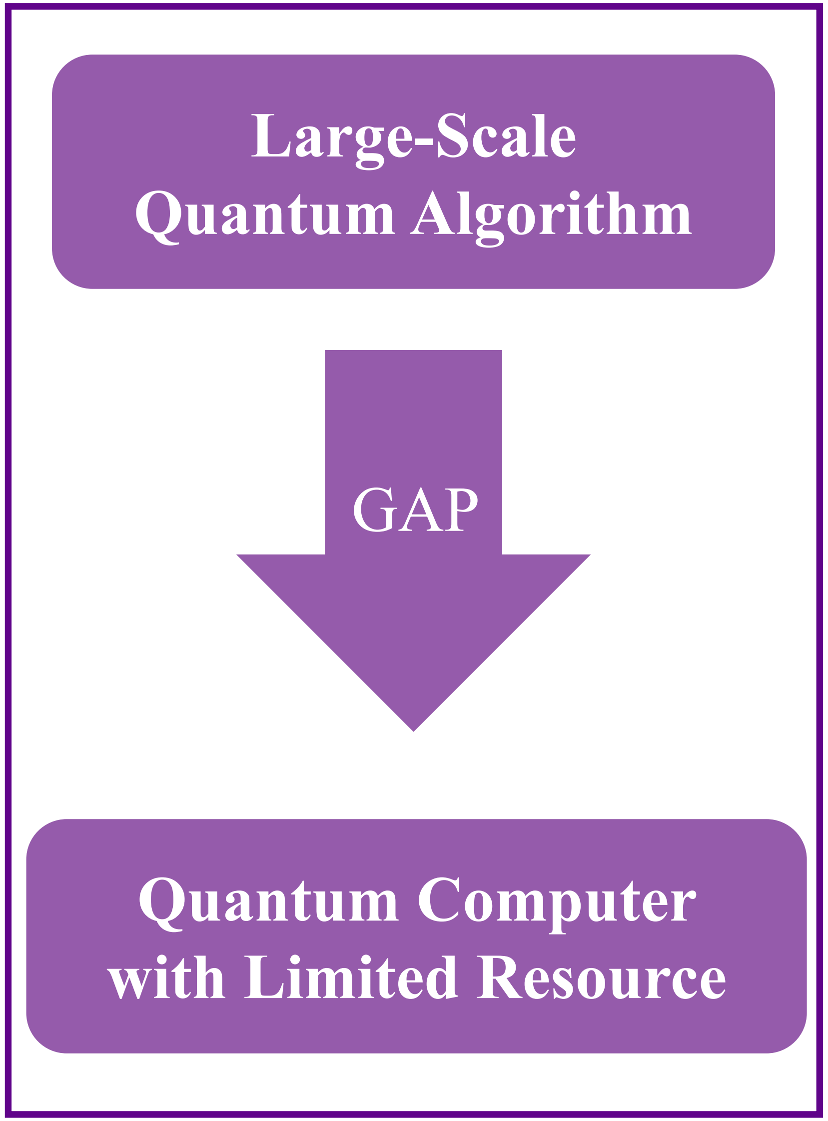


Hard to compile manually for large circuits.

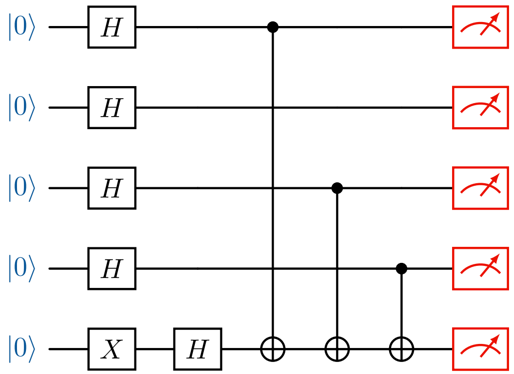
Automatic Tool Needed!

Take-home Message*

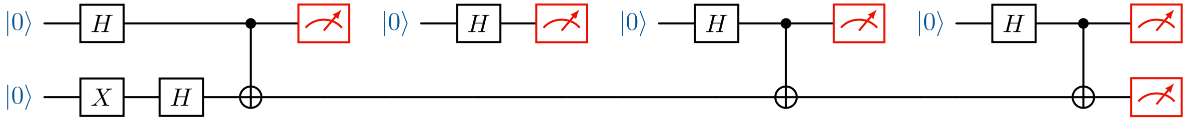


Dynamic Quantum Circuit Compilation

N-qubit Bernstein–Vazirani Algorithm



2-qubit with Dynamic Circuit



Advantages

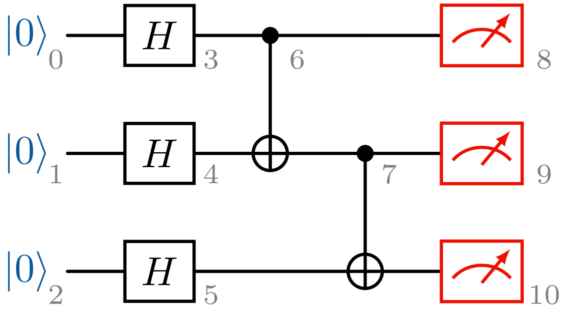
- **Qubit Saving**
- Swap Reduction
- Improved Fidelity

Circuit Compilation via Graph Manipulation*

From Circuit to Graph

Quantum Circuit Representations

Circuit Diagram

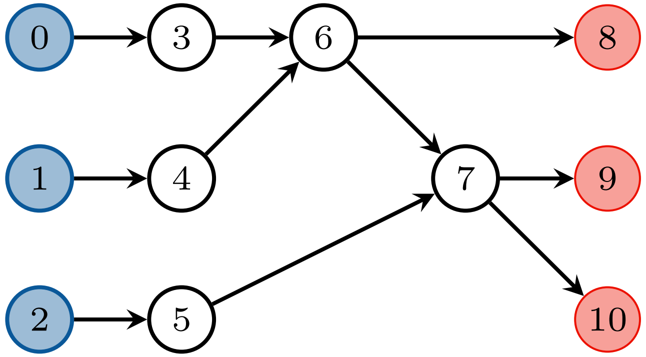


Circuit Instruction

Quantum Circuit Instructions

ID	TYPE	QUBIT	PAR
0	RESET	0	NONE
1	RESET	1	NONE
2	RESET	2	NONE
3	H	0	NONE
4	H	1	NONE
5	H	2	NONE
6	CX	[0, 1]	NONE
7	CX	[1, 2]	NONE
8	MEASURE	0	NONE
9	MEASURE	1	NONE
10	MEASURE	2	NONE

Directed Acyclic Graph (DAG) Representation

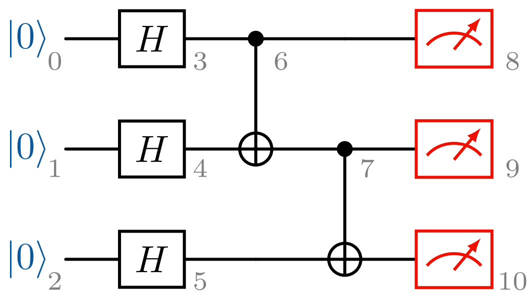


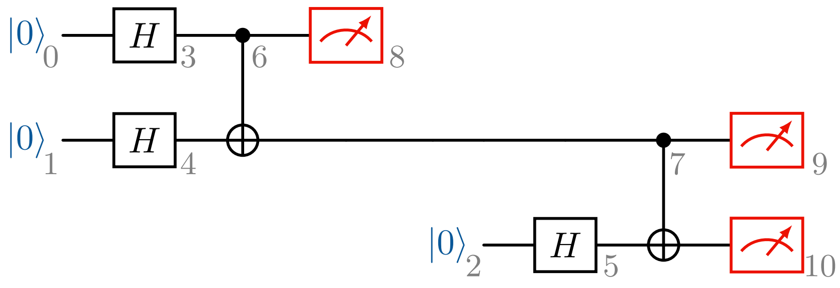
$$G(V = R \cup T \cup I, E)$$

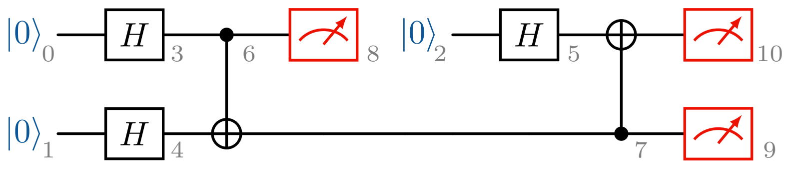
- Root vertices $(r \in R)$, *indegree* $\delta^{-}(r) = 0$
- Terminal vertices $(t \in T)$, *outdegree* $\delta^{+}(t) = 0$
- Internal vertices $(i \in I)$, $\delta^{-}(i) \neq 0, \delta^{+}(i) \neq 0$

Quantum Circuit Compilation via Graph Manipulation*

Dynamic Circuit Compilation

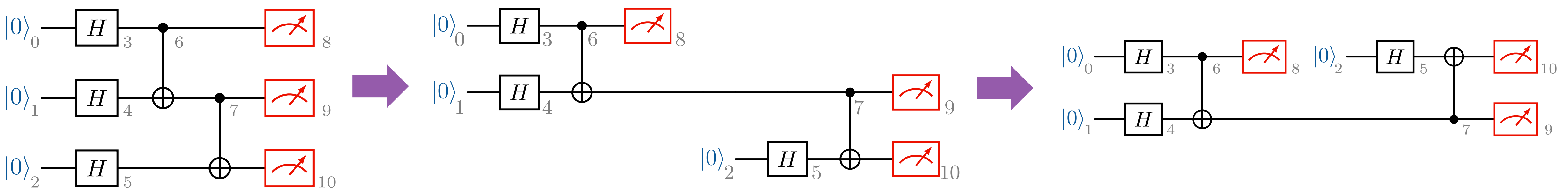




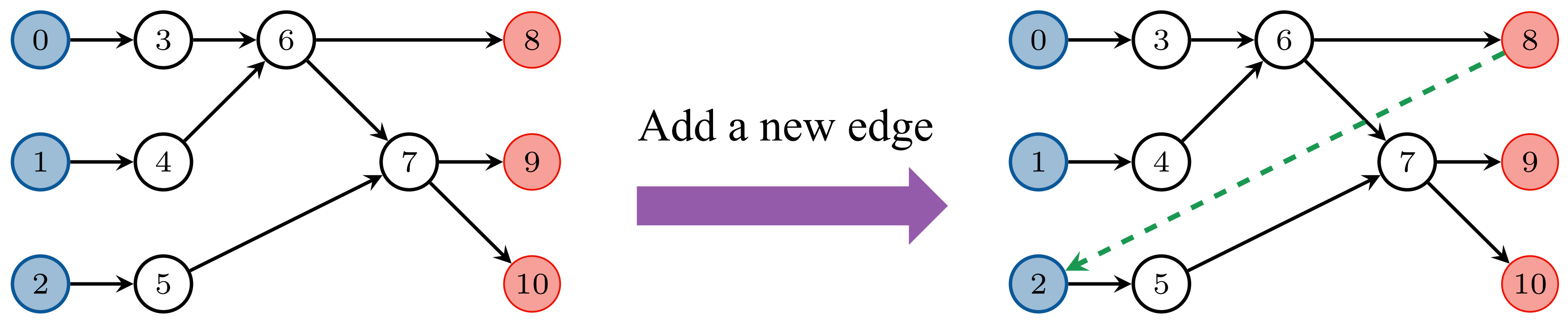


Quantum Circuit Compilation via Graph Manipulation*

Dynamic Circuit Compilation



Graph Manipulation



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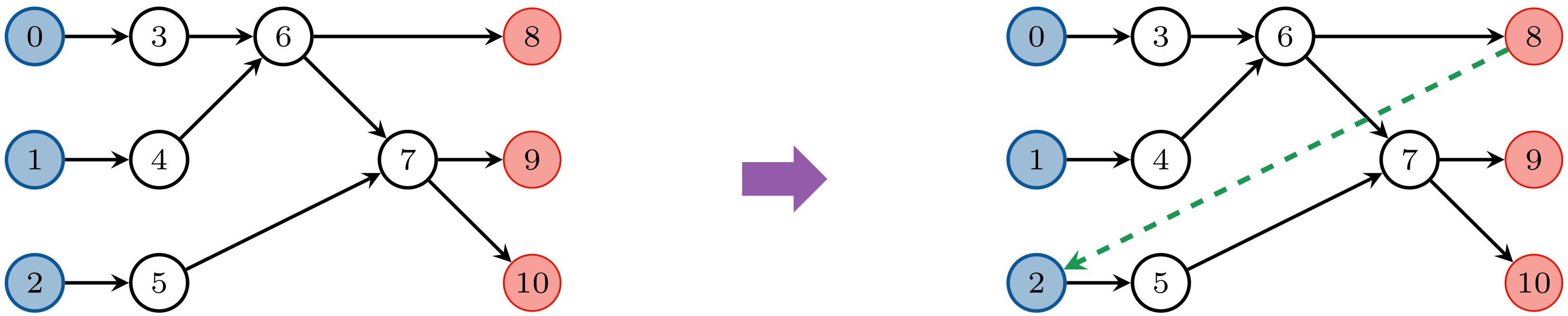

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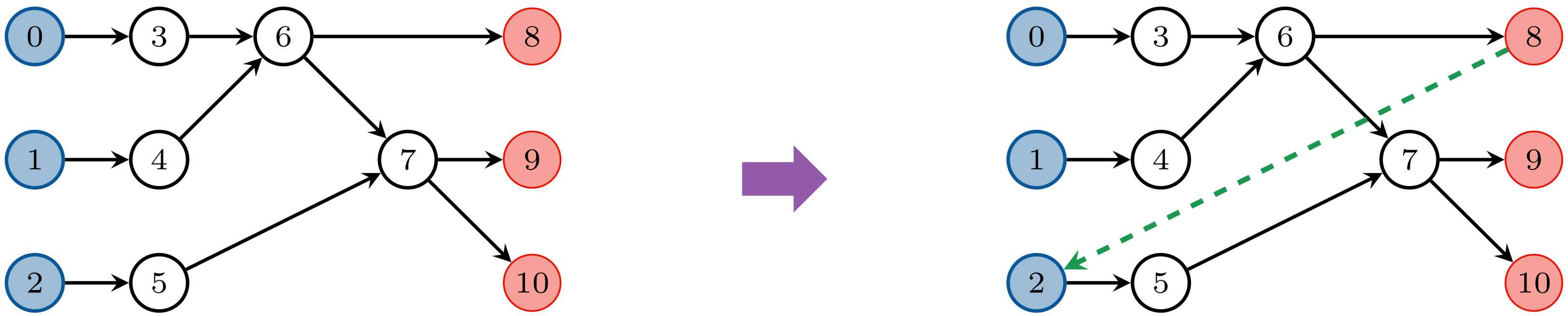


Constraints

- $\forall\, (t, r) \in E'$, *it has* $t \in T$ *and* $r \in R$;
- $\forall\, (t, r) \in E'$, *it has* $outdegree(t) = 1$ *and* $indegree(r) = 1$;
- $G' = (R \cup T, E \cup E')$ *is an* ***acyclic*** *graph.*

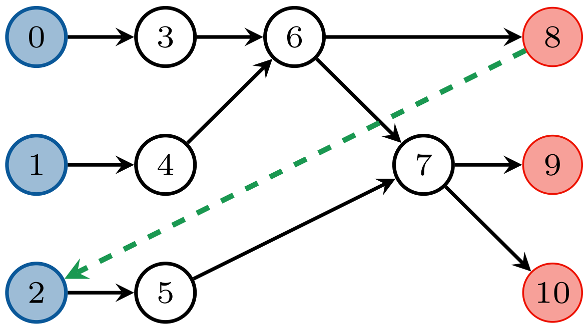
To ensure that the graph manipulation corresponds to a valid circuit compilation

Circuit Compilation via Graph Manipulation*

From Graph to Circuit

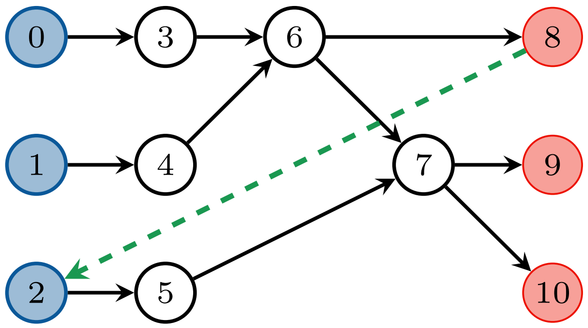
Quantum Circuit Compilation via Graph Manipulation*

DAG with new edges



Quantum Circuit Compilation via Graph Manipulation*

DAG with new edges



Topological

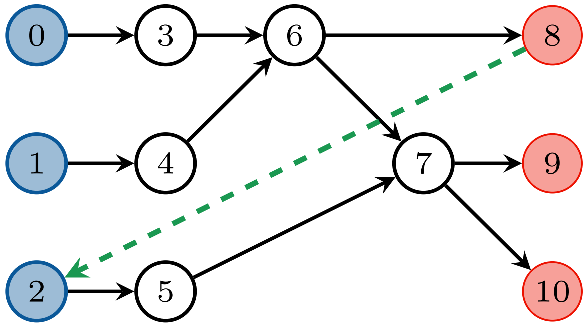
Ordering

Feasible Execution Sequence

$0 \to 1 \to 3 \to 4 \to 6 \to 8 \to 2 \to 5 \to 7 \to 10 \to 9$

Quantum Circuit Compilation via Graph Manipulation*

DAG with new edges



Topological

Ordering

every directed path from *u* to *v*,
u comes before *v* in the ordering

Feasible Execution Sequence

$0 \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 10 \rightarrow 9$

Qubit **Reallocation**

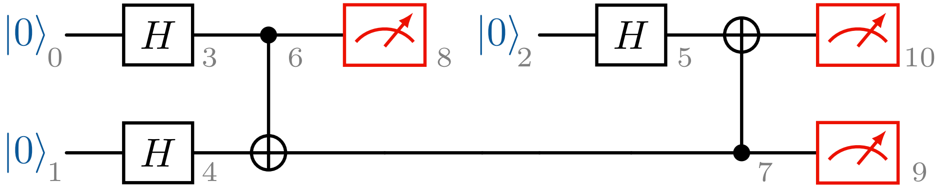
Dynamic Circuit Instructions

Quantum Circuit Instructions

ID	TYPE	QUBIT	PAR
0	RESET	0	NONE
1	RESET	1	NONE
3	H	0	NONE
4	H	1	NONE
6	CX	[0, 1]	NONE
8	MEASURE	0	NONE
2	RESET	0	NONE
5	H	0	NONE
7	CX	[1, 0]	NONE
10	MEASURE	0	NONE
9	MEASURE	1	NONE

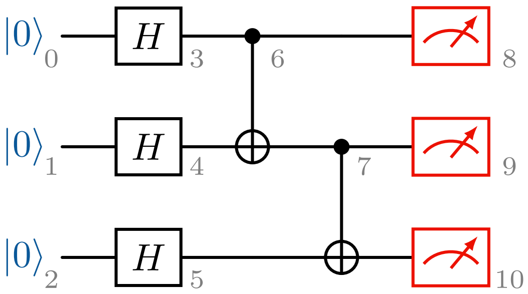
Reallocate 2 to 0

Dynamic Circuit



Compilation Procedures

Static Circuit



Input Circuit

Static Circuit Instruction

Quantum Circuit Instructions

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1	RESET	1	NONE
2	RESET	2	NONE
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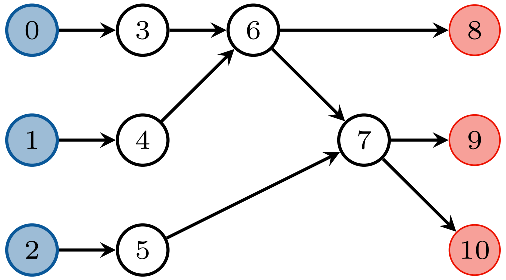
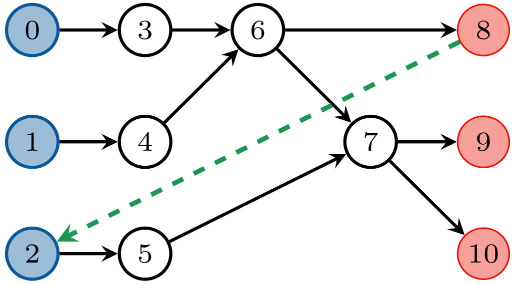
4	H	1	NONE
5	H	2	NONE
6	CX	[0, 1]	NONE
7	CX	[1, 2]	NONE
8	MEASURE	0	NONE
9	MEASURE	1	NONE
10	MEASURE	2	NONE

DAG Representation



DAG with a new edge



Dynamic Circuit Instruction

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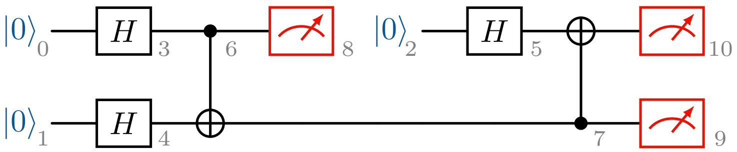
Dynamic Circuit



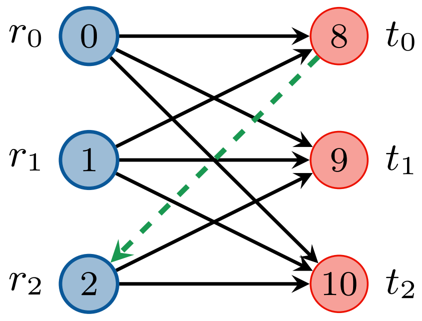
Output Circuit

Optimal Circuit Compilation for Maximizing Qubit Reuse

Optimal Circuit Compilation for Maximizing Qubit Reuse

Optimal Quantum Circuit Compilation as a Binary Integer Programming Problem

Simplified DAG



Biadjacency Matrix

$$\begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \end{array} \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \end{array}$$

Candidate Matrix

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Graph Manipulation

- $\forall\,(t,r) \in E'$, *it has* $t \in T$ *and* $r \in R$;
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- $G' = (R \cup T, E \cup E')$ *is an acyclic graph.*

Exact Characterization

$$\alpha := \max \sum_{i,j=0}^{n-1} F_{ij} \quad \textbf{Maximized added edges}$$

$$\text{s.t. } F_{ij} \le \bar{B}_{ij}, \forall i,j \in \{0,1,\ldots,n-1\}$$

$$\sum_{j=0}^{n-1} F_{ij} \le 1, \forall i \in \{0,1,\ldots,n-1\}$$

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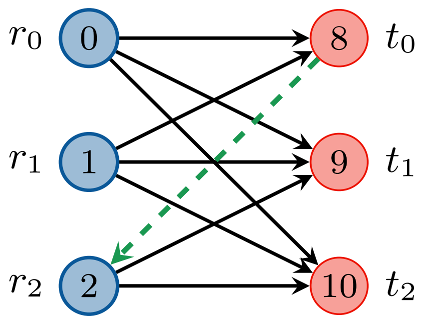
$$\begin{pmatrix} O_n & B \\ F & O_n \end{pmatrix} \quad \textit{nilpotent}$$

Optimal Circuit Compilation for Maximizing Qubit Reuse

Optimal Quantum Circuit Compilation as a Binary Integer Programming Problem

Simplified DAG



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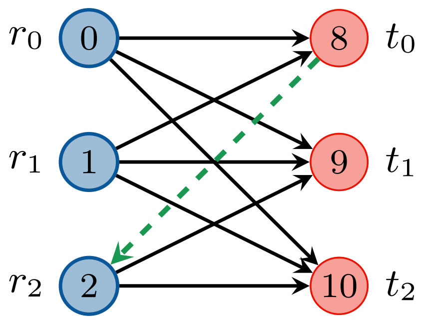
Optimal Quantum Circuit Compilation as a Binary Integer Programming Problem

Simplified DAG



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$$\begin{cases} \displaystyle\sum_{j=0}^{n-1} F_{ij} \le 1, \forall i \in \{0,1,\ldots,n-1\} \\ \displaystyle\sum_{i=0}^{n-1} F_{ij} \le 1, \forall j \in \{0,1,\ldots,n-1\} \end{cases}$$

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Optimal Circuit Compilation for Maximizing Qubit Reuse

Optimal Quantum Circuit Compilation as a Binary Integer Programming Problem

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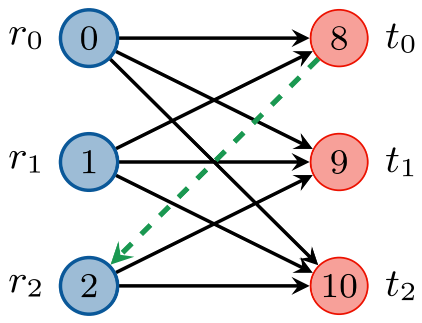

Biadjacency Matrix

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\begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\
\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}
\end{array}
$$

Candidate Matrix

$$
\begin{array}{c} \\ t_0 \\ t_1 \\ t_2 \end{array}
\begin{array}{c}
\begin{array}{ccc} r_0 & r_1 & r_2 \end{array} \\
\begin{pmatrix} 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{array}
$$

Graph Manipulation

- $\forall\, (t, r) \in E'$, *it has* $t \in T$ *and* $r \in R$;
- $\forall\, (t, r) \in E'$, *it has* $outdegree(t) = 1$ *and* $indegree(r) = 1$;
- $G' = (R \cup T, E \cup E')$ *is an acyclic graph.*

Exact Characterization

$$\alpha := \max \sum_{i,j=0}^{n-1} F_{ij} \quad \textbf{Maximized added edges}$$

$$\text{s.t. } F_{ij} \le \bar{B}_{ij}, \forall i, j \in \{0, 1, \dots, n-1\}$$

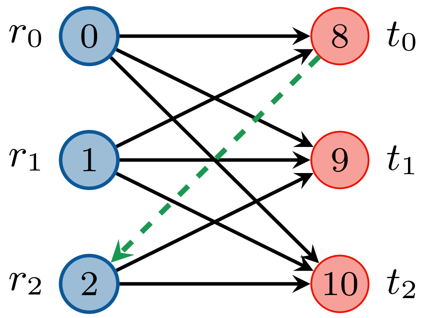
$$\begin{cases} \displaystyle\sum_{j=0}^{n-1} F_{ij} \le 1, \forall i \in \{0, 1, \dots, n-1\} \\ \displaystyle\sum_{i=0}^{n-1} F_{ij} \le 1, \forall j \in \{0, 1, \dots, n-1\} \end{cases}$$

$$\begin{pmatrix} O_n & B \\ F & O_n \end{pmatrix} \quad \textit{nilpotent}$$

Optimal Circuit Compilation for Maximizing Qubit Reuse

Optimal Quantum Circuit Compilation as a Binary Integer Programming Problem

Simplified DAG



Biadjacency Matrix

$$\begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \end{array} \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \end{array}$$

Candidate Matrix

$$\begin{array}{c} \\ t_0 \\ t_1 \\ t_2 \end{array} \begin{array}{c} \begin{array}{ccc} r_0 & r_1 & r_2 \end{array} \\ \begin{pmatrix} 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Graph Manipulation

- $\forall\,(t,r) \in E'$, *it has* $t \in T$ *and* $r \in R$;
- $\forall\,(t,r) \in E'$, *it has* $outdegree(t) = 1$ *and* $indegree(r) = 1$;
- $G' = (R \cup T, E \cup E')$ *is an acyclic graph.*

Exact Characterization

$$\alpha \coloneqq \max \sum_{i,j=0}^{n-1} F_{ij}$$

NP-hard [*]

$$\text{s.t. } F_{ij} \le \bar{B}_{ij}, \forall i,j \in \{0,1,\dots,n-1\}$$

$$\begin{cases} \sum_{j=0}^{n-1} F_{ij} \le 1, \forall i \in \{0,1,\dots,n-1\} \\ \sum_{i=0}^{n-1} F_{ij} \le 1, \forall j \in \{0,1,\dots,n-1\} \end{cases}$$

$$\begin{pmatrix} O_n & B \\ F & O_n \end{pmatrix} \quad \textit{nilpotent}$$

[1] Hanru Jiang. 2024. Qubit Recycling Revisited. Proc. ACM Program. Lang. 8, PLDI, Article 198 (June 2024), 24 pages.

[*] The author in [1] proved that an equivalent optimization problem is NP-hard.

Optimal Circuit Compilation for Maximizing Qubit Reuse

Summary of Explored Circuits

	Quantum circuits	Original	Compiled
	Bernstein-Vazirani algorithm	$n+1$	2 (1)
$\star$	Quantum Fourier transform	n	n
$\star$	Quantum phase estimation	n	n
$\star$	Shor's algorithm	n	n
	Grover's algorithm	n	n
	Quantum counting algorithm	n	n
	Quantum ripple carry adder circuit	n (4)	4 (3)
	Linearly entangled circuit with l layers	n	$l+1$
	Circularly entangled circuit with l layers	n	3
	Pairwisely entangled circuit with l layers	n	$2l+1$
	Fully entangled circuit	n	n
	Diamond-structured quantum circuit	$2n$	$n+1$
	MBQC with cluster state of size (w, d)	wd	$w+1$
	MBQC with brickwork state of size (w, d)	wd	$w+1$

Cases in red indicates the compiled width is **optimal** and cases in blue indicates **irreducible circuit**.

$\star$ **Depends on specific circuit implementation of the algorithm**

Refer to the full paper for more details [arXiv: 2310.1102]

Heuristic Algorithms

How to add as many edges as possible?

Heuristic Algorithm: Greedy Algorithms*

Algorithm: Greedy Algorithm

Greedy Algorithm

Makes a **locally optimal** choice at each step → **Maximize** the possibility to add more edges in subsequent steps

Heuristic Algorithm: Greedy Algorithms*

Algorithm: Greedy Algorithm

Greedy Algorithm

Makes a **locally optimal** choice at each step → **Maximize** the possibility to add more edges in subsequent steps

Procedure

- **Temporarily** integrates the edge into the simplified DAG and update the candidate matrix
- Score the candidate edge as the **summation** of all elements within the updated candidate matrix
- **Randomly** select a candidate edge with the **highest score**

Heuristic Algorithm: Greedy Algorithms*

Algorithm: Greedy Algorithm

Worst-case Time Complexity $O(mn + n^5)$

n: input circuit width, m: # of instructions

---

e.g. 5-qubit Bernstein-Vazirani

Edge under evaluation

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & \boxed{1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Update candidate matrix

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ \boxed{\mathbf{0}} & 0 & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & \boxed{\mathbf{0}} & 1 & 1 & 0 \\ 0 & \boxed{\mathbf{0}} & 0 & 0 & 0 \\ 0 & \boxed{\mathbf{0}} & 0 & 1 & 0 \\ \boxed{\mathbf{0}} & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} \\ 0 & \boxed{\mathbf{0}} & 0 & 0 & 0 \end{pmatrix}$$

Score matrix

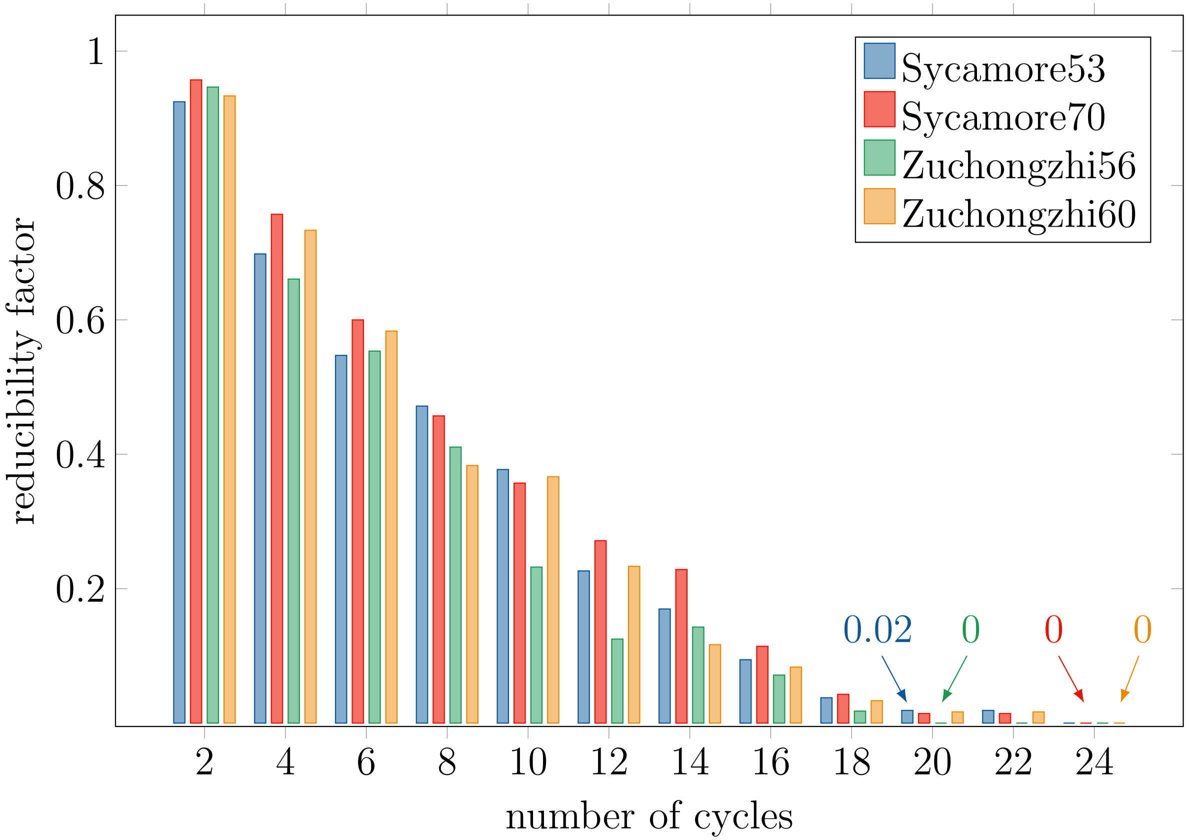
$$\begin{pmatrix} 0 & 3 & \boxed{\mathbf{7}} & 6 & 0 \\ 3 & 0 & 3 & 3 & 3 \\ 0 & 3 & 0 & \boxed{\mathbf{7}} & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \end{pmatrix}$$

Numerical Evaluation

Performance of the heuristic algorithms?

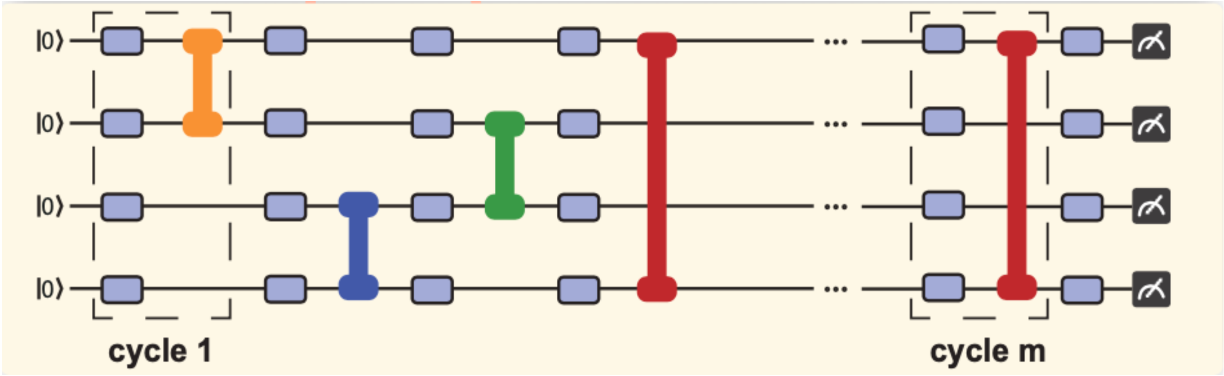
Numerical Evaluation: Quantum Supremacy Circuits

*Demonstrating Quantum Supremacy



Reducibility Factor $r = 1 - \frac{\textit{compiled width}}{\textit{original width}} \in [0, 1)$

The larger, the better



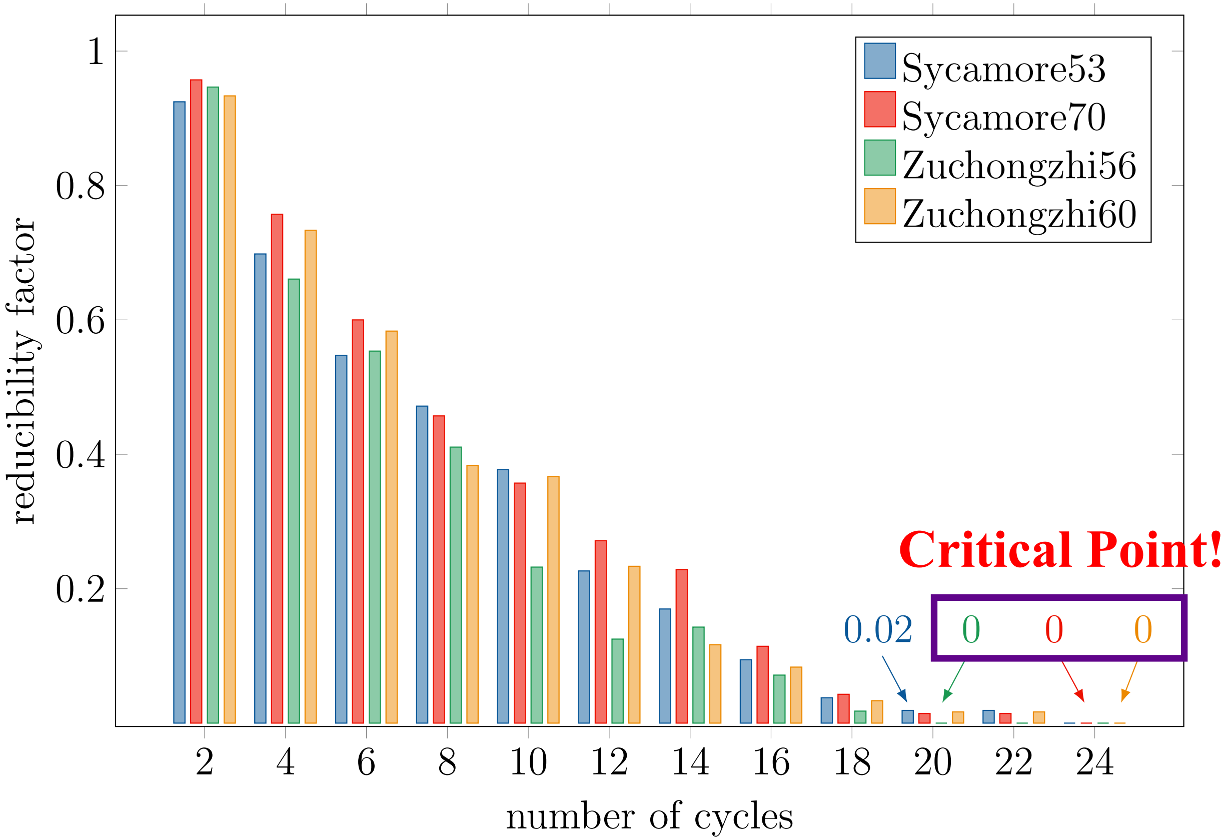
The number of cycles cannot be too large or too small.

How to determine this number?

[1] Frank Arute et al. Quantum supremacy using a programmable superconducting processor. *Nature* 574, 505-510 (2019).
[2] Alexis Morvan et al. Phase transition in Random Circuit Sampling. arXiv:2304.11119 (2023).
[3] Yulin Wu et al. Strong quantum computational advantage using a superconducting quantum processor. *Phys. Rev. Lett.* 127, 180501 (2021).
[4] Qingling Zhu et al. Quantum computational advantage via 60-qubit 24-cycle random circuit sampling. Science Bulletin 67, 240-245 (2022).

Numerical Evaluation: Quantum Supremacy Circuits

*Demonstrating Quantum Supremacy



Interesting Observation I

Sycamore 70

Zuchongzhi 56

Zuchongzhi 60

Irreducible

Quantum Circuit

The number of cycles cannot be too large or too small.

How to determine this number?

[1] Frank Arute et al. Quantum supremacy using a programmable superconducting processor. *Nature* 574, 505-510 (2019).
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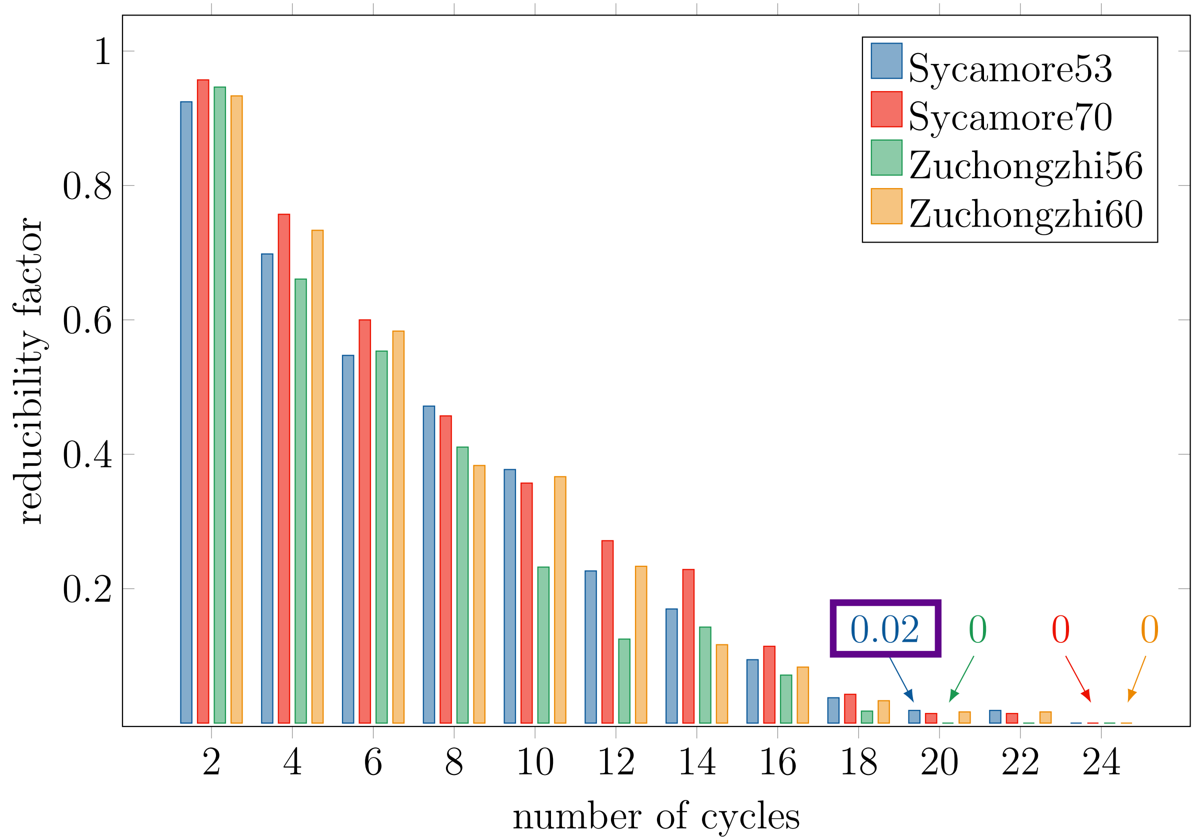
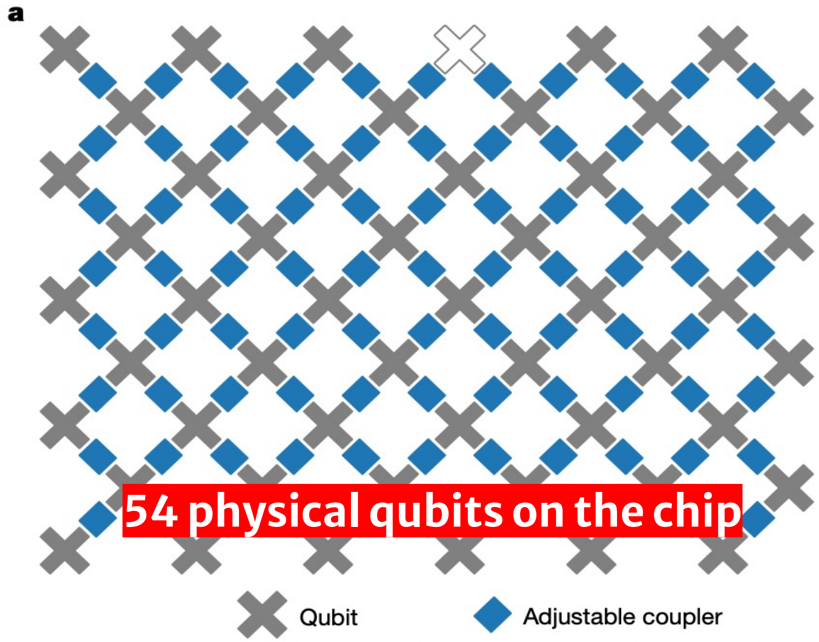
Numerical Evaluation: Quantum Supremacy Circuits

*Demonstrating Quantum Supremacy



Interesting Observation II

Sycamore 53 → $r = 0.02$



[1] Frank Arute et al. Quantum supremacy using a programmable superconducting processor. *Nature* 574, 505-510 (2019).
[2] Alexis Morvan et al. Phase transition in Random Circuit Sampling. arXiv:2304.11119 (2023).
[3] Yulin Wu et al. Strong quantum computational advantage using a superconducting quantum processor. *Phys. Rev. Lett*. 127, 180501 (2021).
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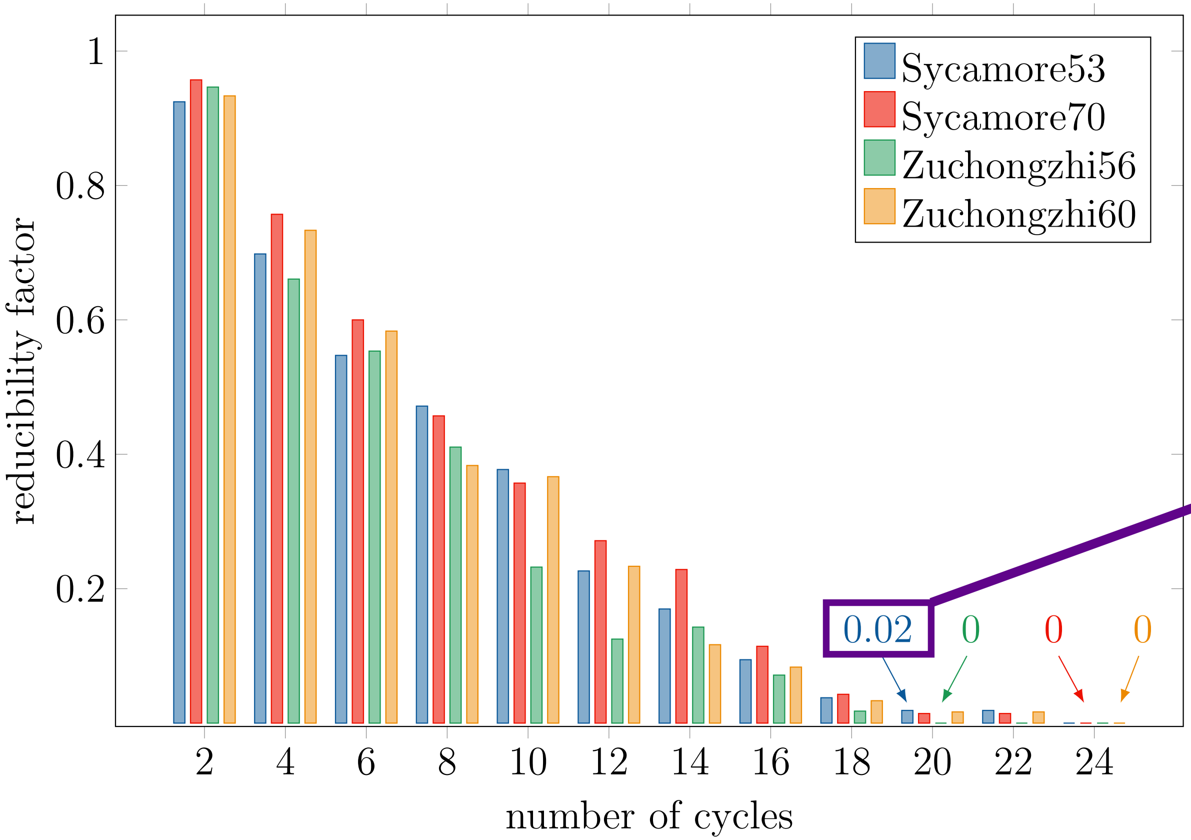
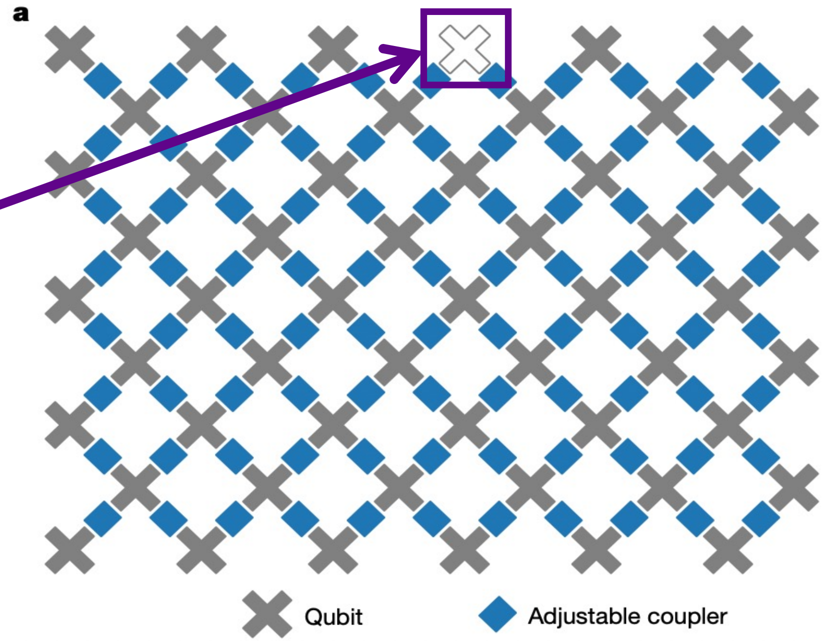
Numerical Evaluation: Quantum Supremacy Circuits

*Demonstrating Quantum Supremacy



Interesting Observation II

Sycamore 53 → $r = 0.02$



[1] Frank Arute et al. Quantum supremacy using a programmable superconducting processor. *Nature* 574, 505-510 (2019).
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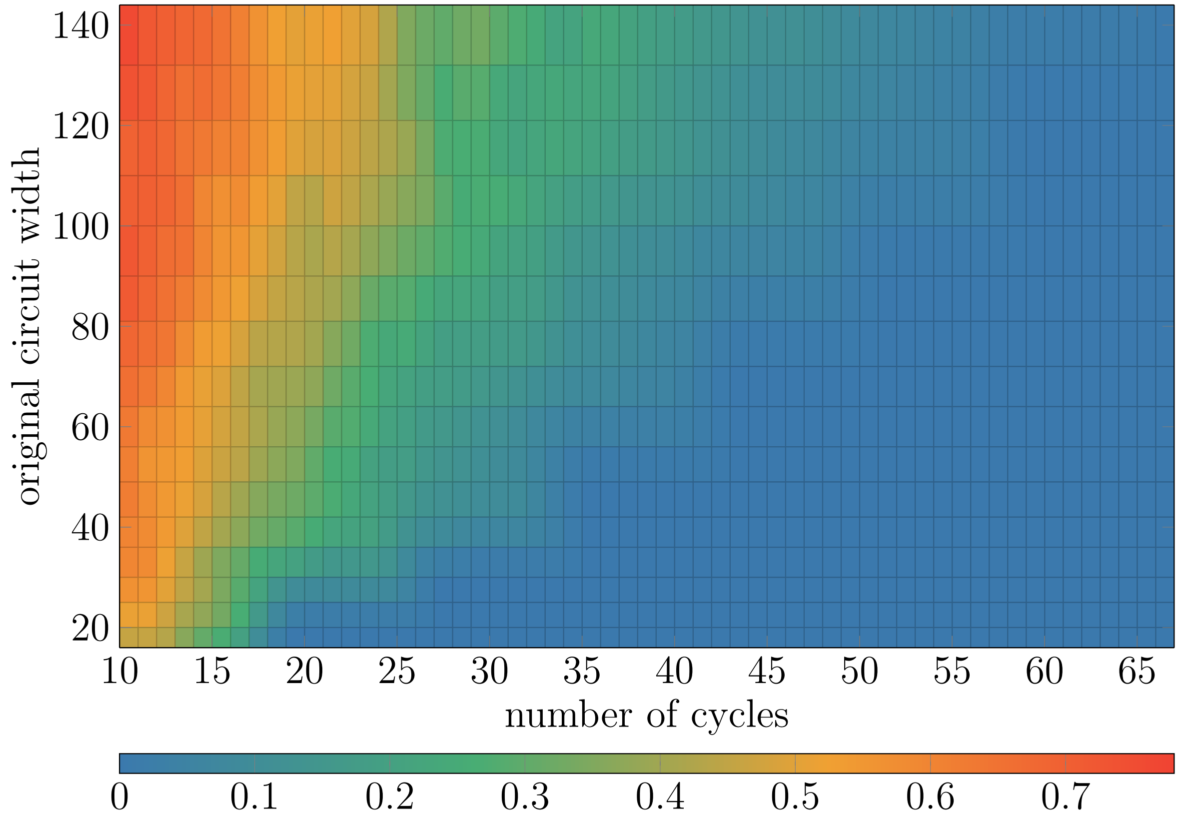
[4] Qingling Zhu et al. Quantum computational advantage via 60-qubit 24-cycle random circuit sampling. Science Bulletin 67, 240-245 (2022).

Numerical Evaluation: Quantum Supremacy Circuits

Google Random Circuit Sampling (GRCS)



Reducibility Factor

The larger the number of cycles

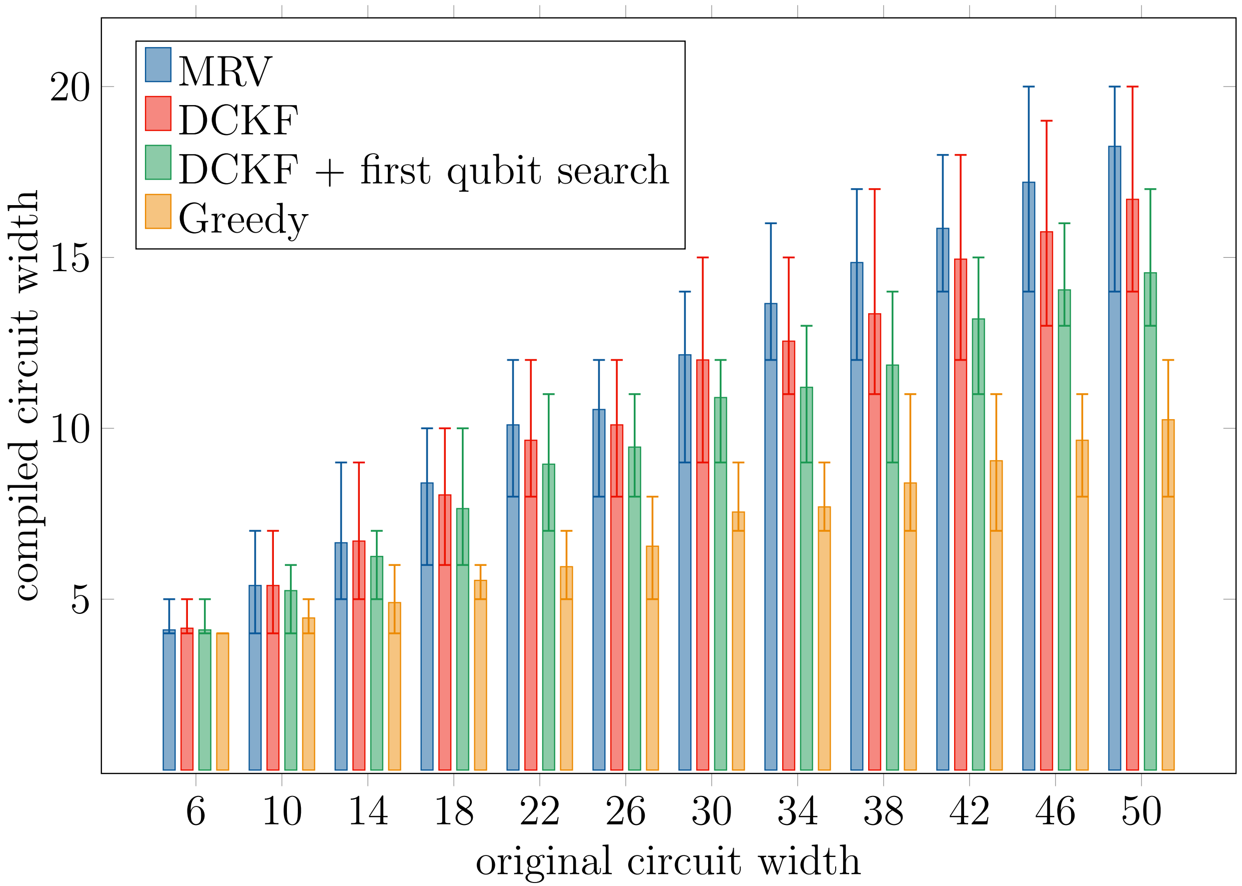
The less reducible the circuit

[1] Sergio Boixo et al. Characterizing quantum supremacy in near-term devices. *Nature Phys* **14**, 595–600 (2018).
[2] GRCS. https://github.com/sboixo/GRCS

Numerical Evaluation: QAOA circuits

QAOA circuits for max-cut problem on random 3-regular graph

Commutable ZZ gates

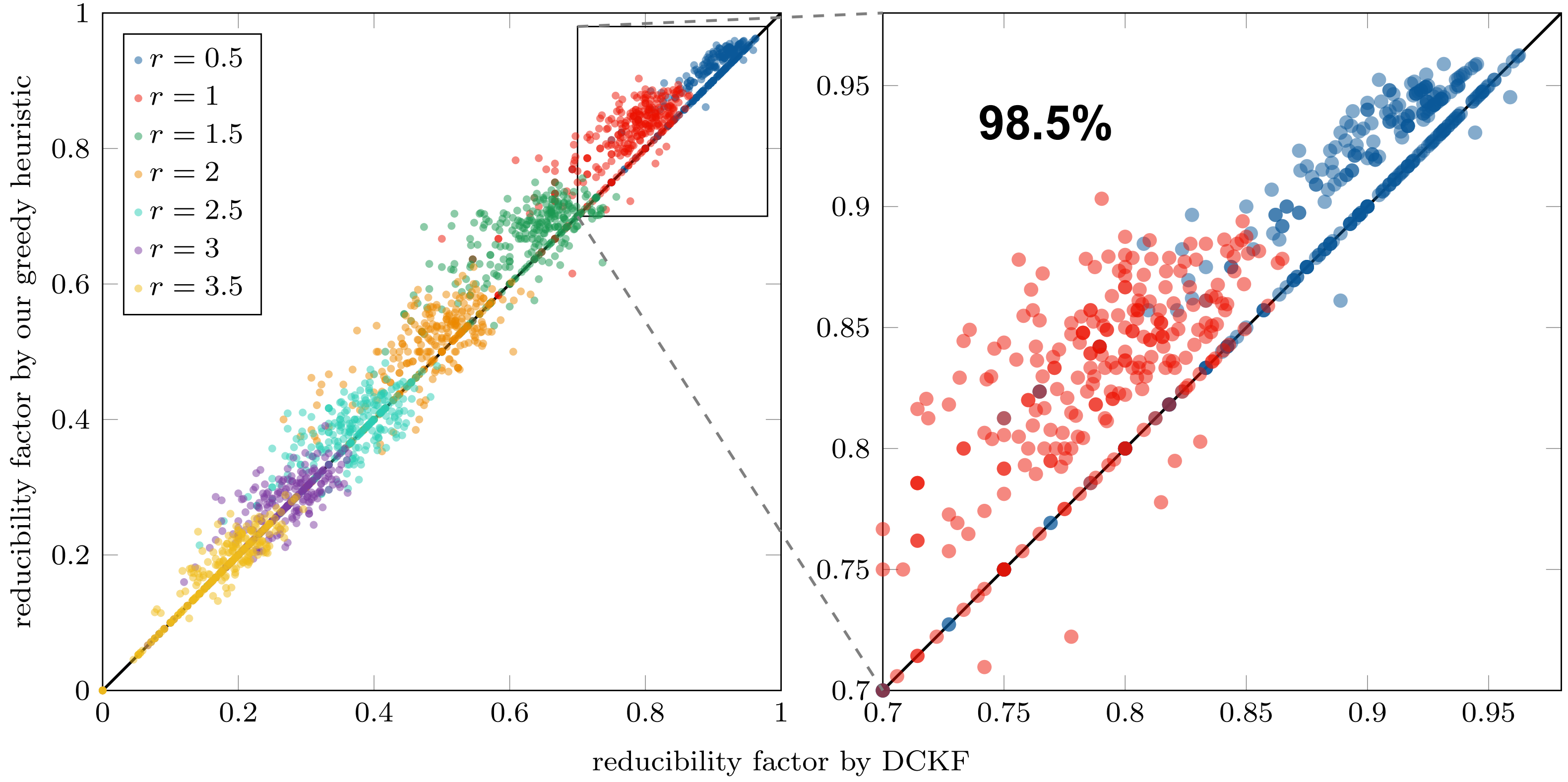


Our methods account for **commutable gates**, which leads to better performance.

[1] Matthew DeCross et al. Qubit-reuse compilation with mid-circuit measurement and reset. arXiv:2210.08039 (2022).

Numerical Evaluation: Random Quantum Circuits

Normal Random Circuit



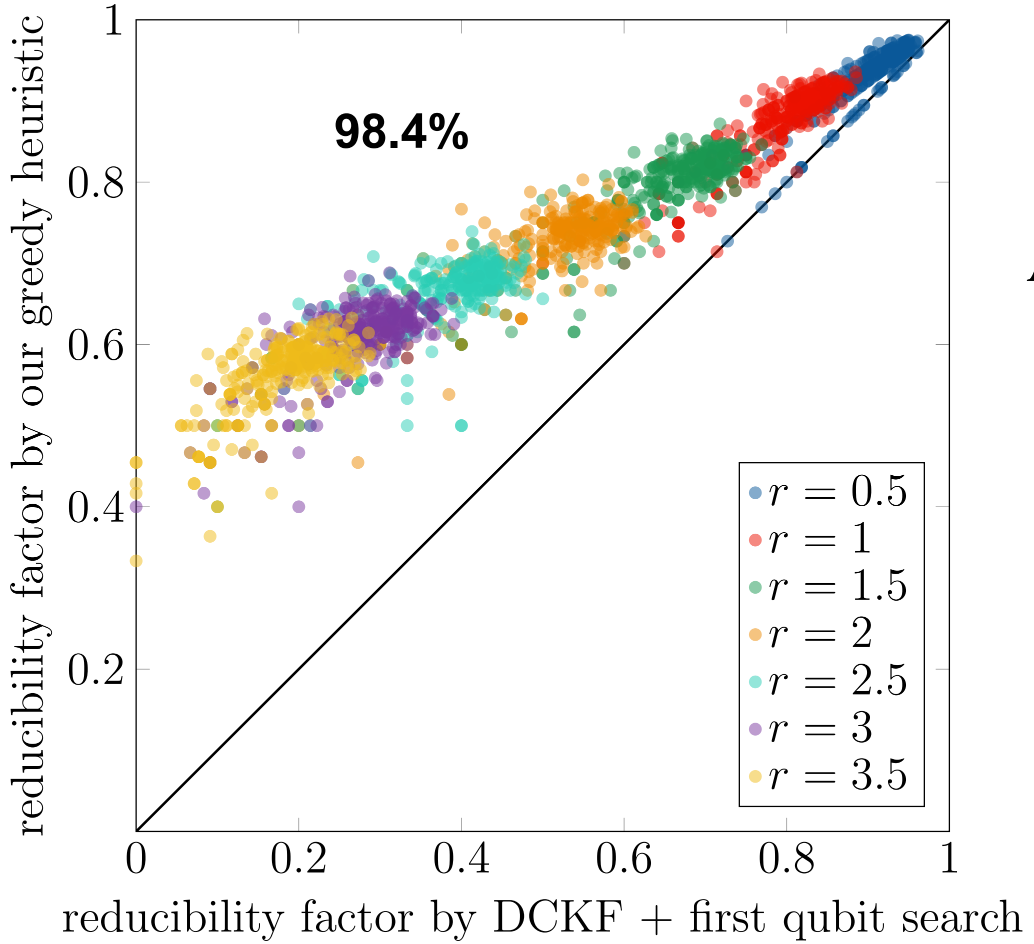
$$r = \frac{\#\ of\ two-qubit\ gates}{\#\ of\ qubits}$$

Outperforms in approximately **87.6% of cases**, with a **strict advantage** over **49.6%** of random circuits

[1] Matthew DeCross et al. Qubit-reuse compilation with mid-circuit measurement and reset. arXiv:2210.08039 (2022).

Numerical Evaluation: Random Quantum Circuits

Random IQP Circuit



A lot of commutable gates

Outperforms in nearly **100% of instances**

[1] Matthew DeCross et al. Qubit-reuse compilation with mid-circuit measurement and reset. arXiv:2210.08039 (2022).

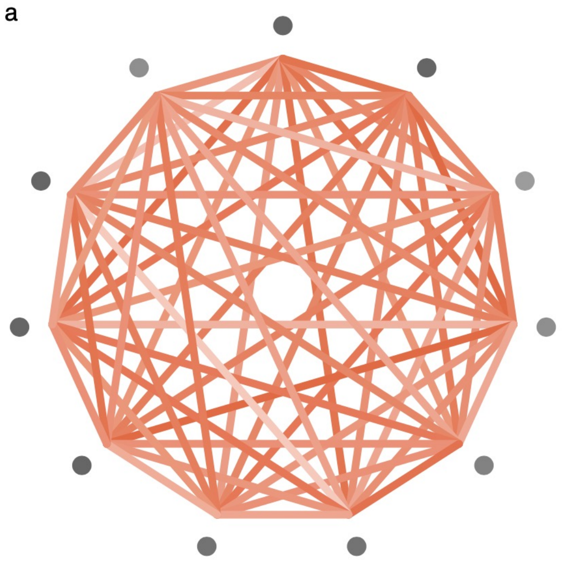
Numerical Evaluation: Noisy Simulation

Noisy Simulation of a 11-qubit Bernstein-Vazirani Algorithm

Compilation reduces the number of qubits required,

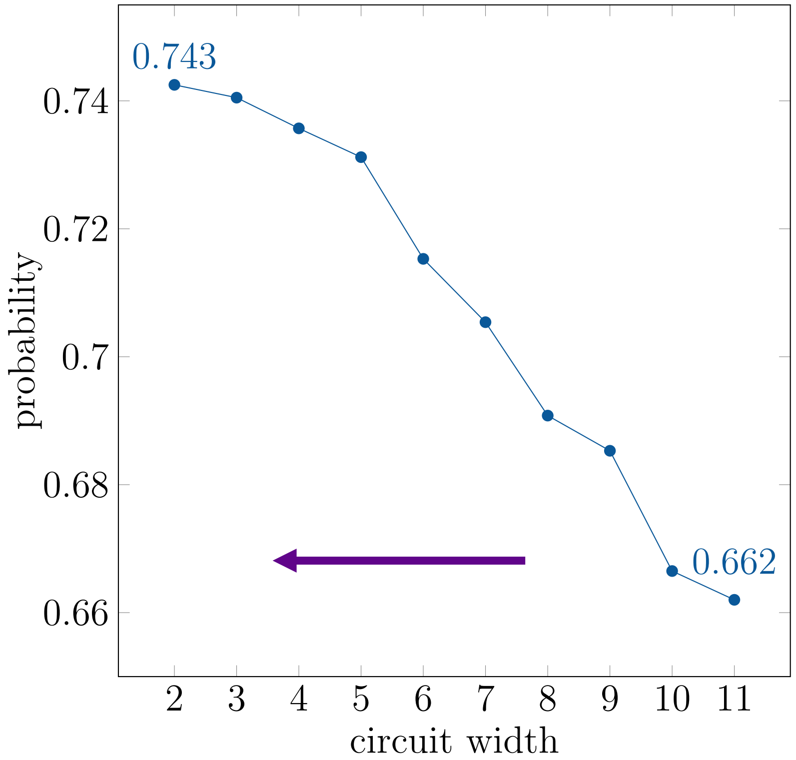
allowing us to choose physical qubits with better performance.

11-qubit Trapped-Ion Quantum Computer[1]



All-to-all connectivity

Experiment Results

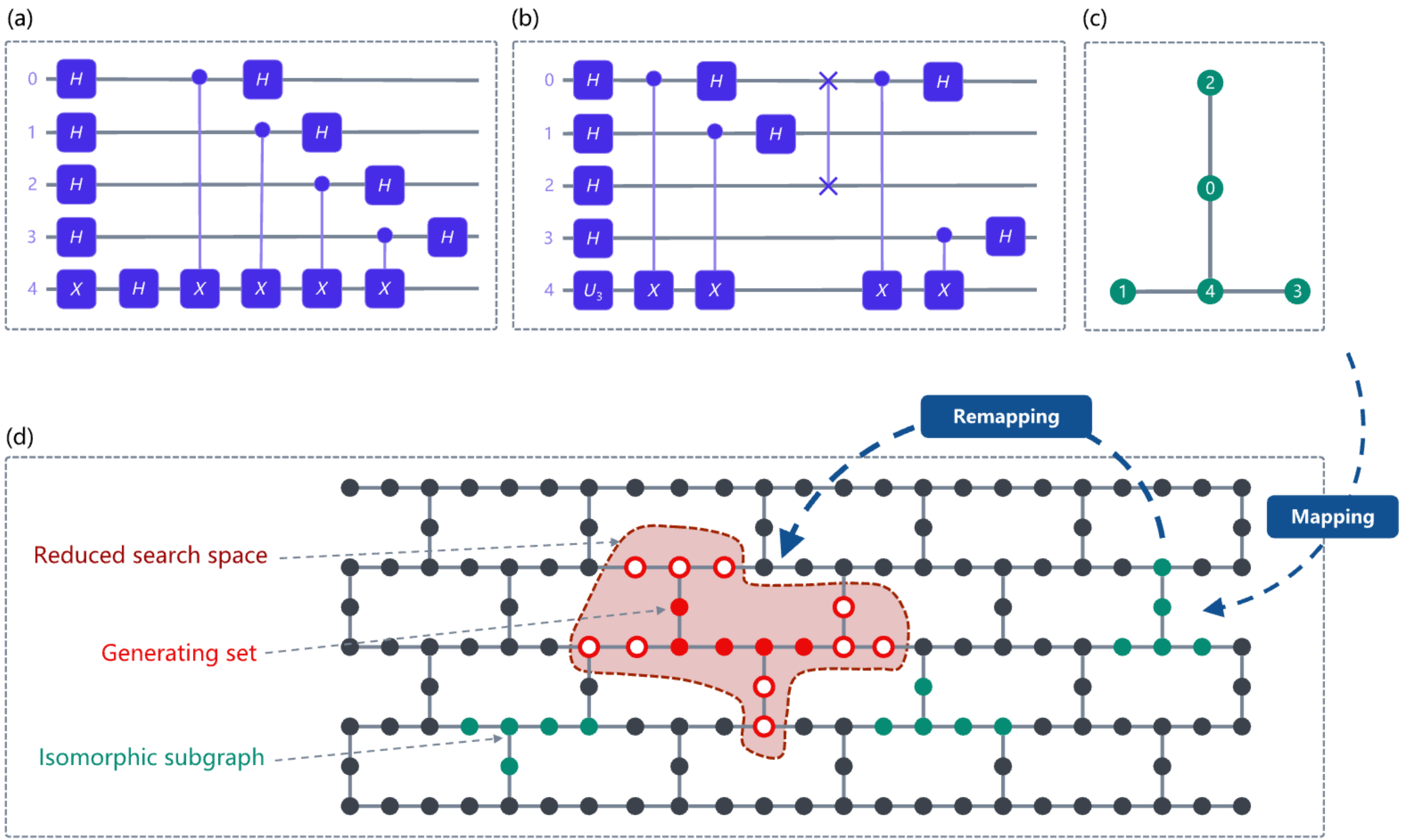


Qubit reduce up to 82% and probability improve 9%

[1] K. Wright et al. Benchmarking an 11-qubit quantum computer. *Nature Communications* 10, 5464 (2019).

Quantum circuit remapping using symmetry

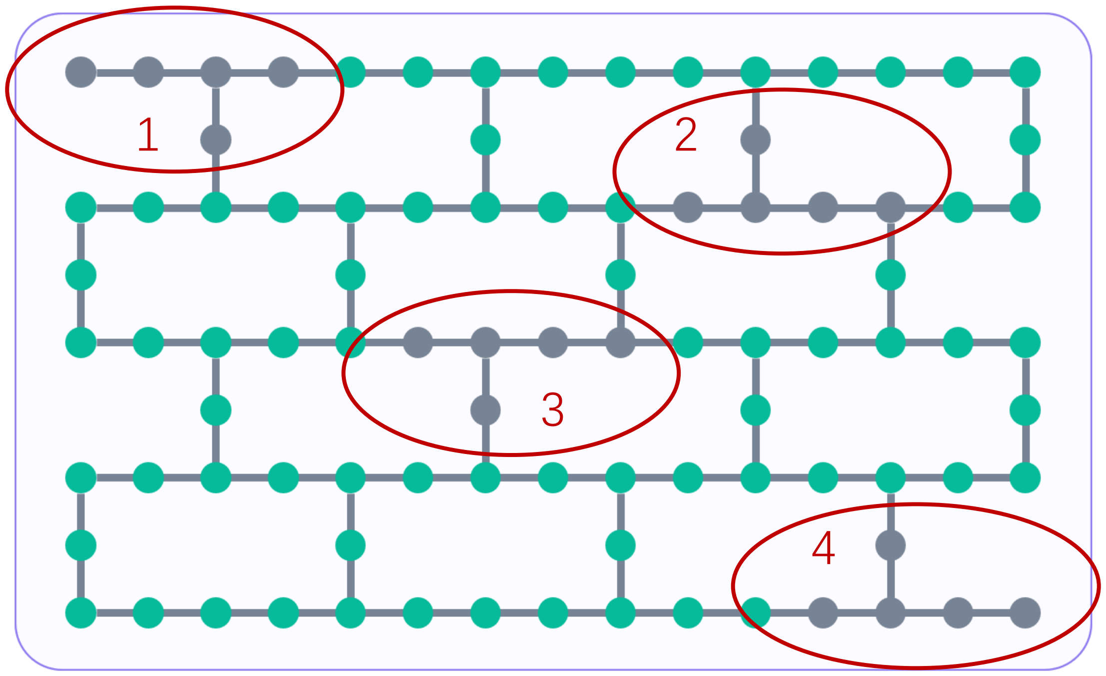
Quantum circuit mapping and remapping



Qubit mapping: determine which physical qubits will be used for executing a logical quantum circuit.

Subgraph Matching Problem

- In a real quantum system, each qubit has different performance, some good and some bad.
- **How to find the best set of qubits to implement your circuit?**
- **Find all isomorphic graphs**



Coupling graph of a quantum chip

Subgraph matching problem

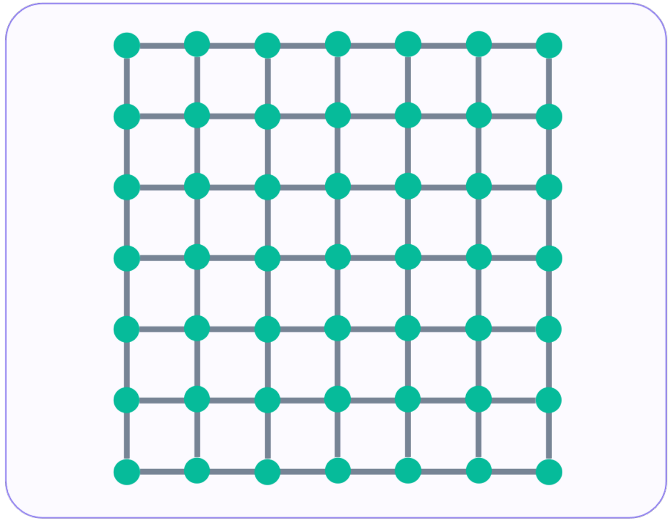
Given a graph G and a graph T, what are all subgraphs in G that are isomorphic to T?

(Stephen A. Cook, 1971) Subgraph matching problem is **NP-complete in general**.

But the **good news** is that the coupling graph is **symmetric**

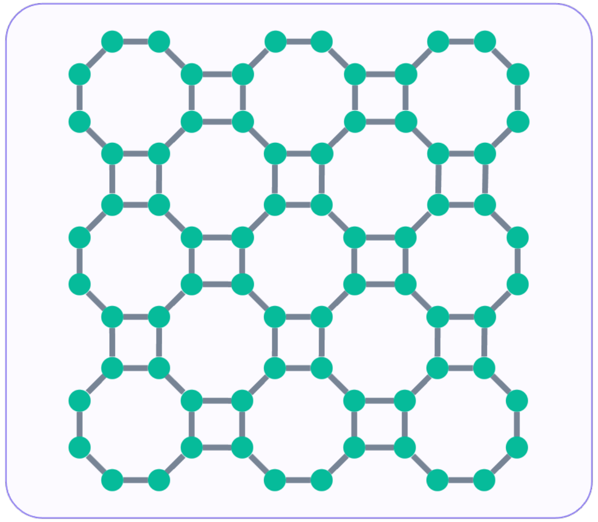
Common quantum hardware structures and their symmetries

Qubit connectivity of some representative quantum computing systems



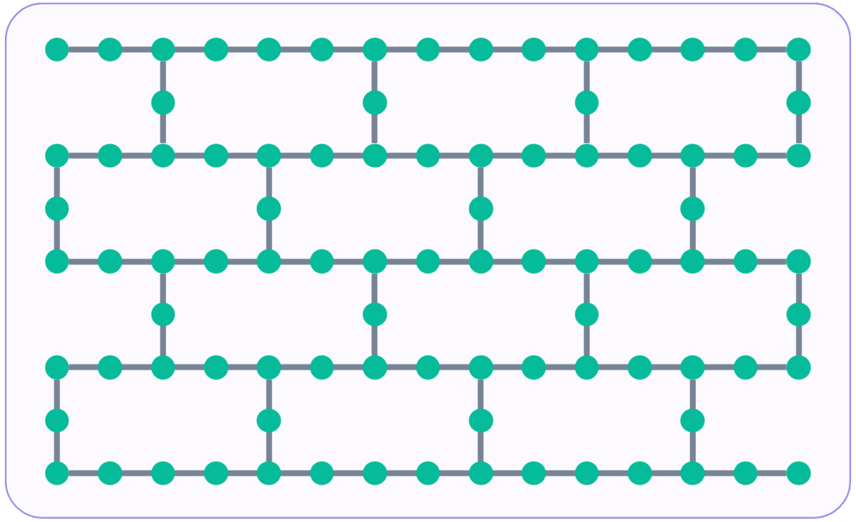
(a) 2D grid

Example: Google Sycamore



(b) Octagonal

Example: Rigetti



(c) Heavy-hex

Example: IBM Falcon

The hardware structure exhibits translation symmetry, rotation symmetry, inversion symmetry⋯

Symmetry-based quantum circuit mapping: algorithm

We have shown that, in many practical cases, this algorithm has a time complexity of $O(n)$ with n the number of qubits in the hardware, which is optimal for the subgraph matching problem.

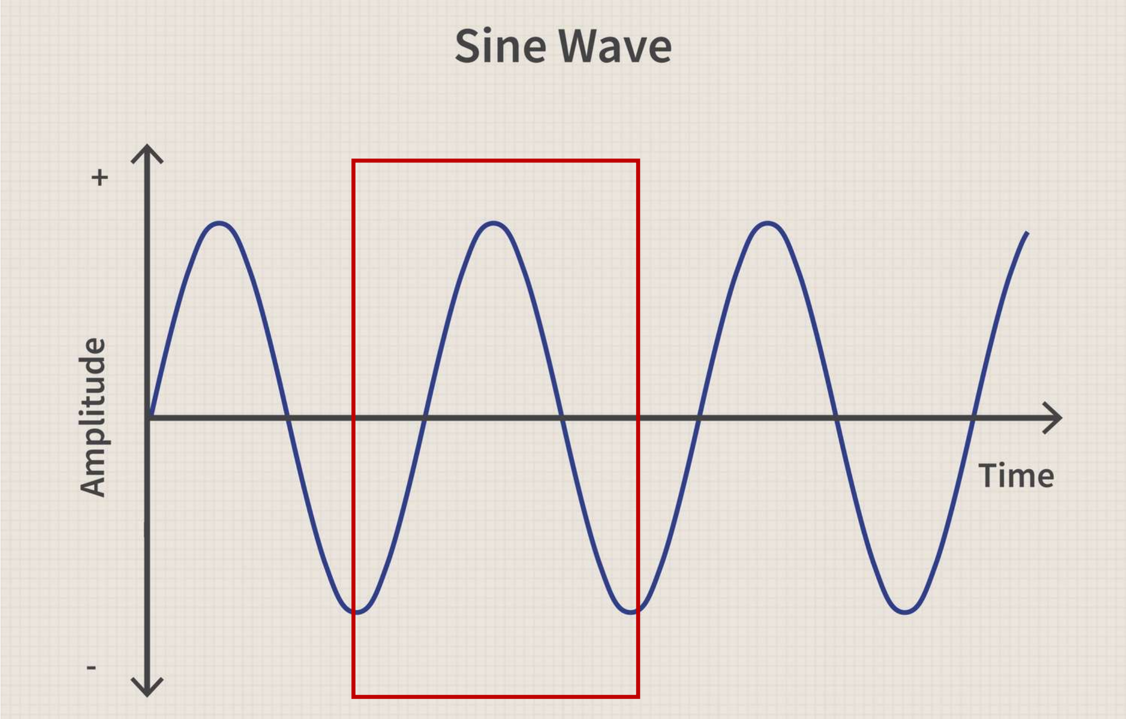
ALGORITHM 1. Symmetry-based subgraph matching

Input : A pattern graph G, a target graph T, a group of automorphisms F of the target graph and the associated generating set S.

Output: All subgraph isomorphisms of G in T.

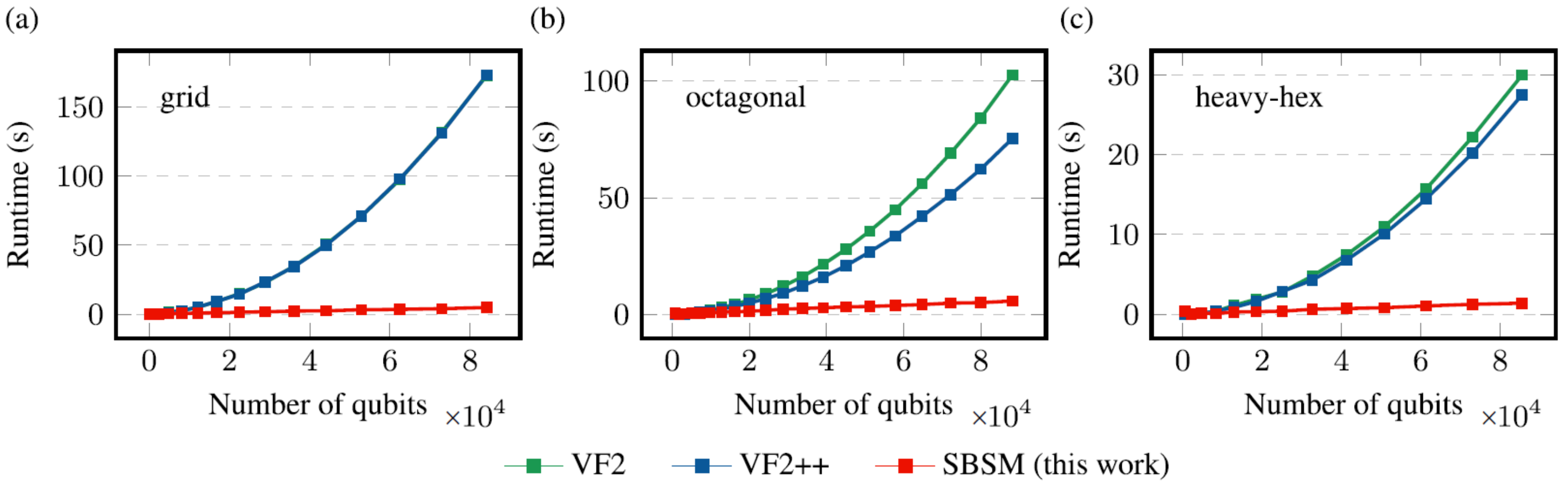
Let r be the radius of the pattern graph G;
Let $N_T^r(S)$ be the r-th order neighborhood of S;
Let R be the induced subgraph of $N_T^r(S)$ in T;
Obtain the set of all isomorphic graphs of G within R and denote this set as H_0;
Let H be an empty list;
for $G' \in H_0$ **do**
  Let $M = \{\tilde{f}(G') : f \in F, \tilde{f}(G') \subseteq T\}$;
  Append M to H;
end
Return H;

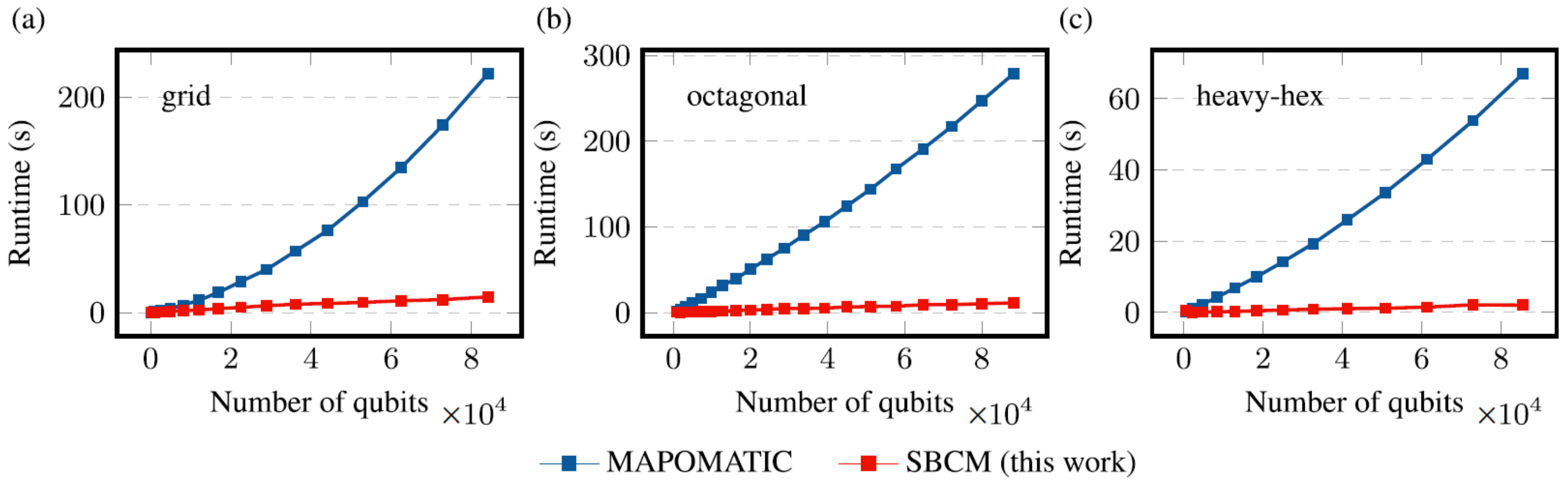
Algorithm	Time complexity
VF2	$O(n^2) \sim O(n!\,n)$
SBSM (this work)	$\mathbf{O(n)}$



Finding all the maxima = find all maxima in one period

Numerical comparison





IBM qiskit

Nation, Paul D., and Matthew Treinish. "Suppressing quantum circuit errors due to system variability." PRX Quantum 4.1 (2023): 010327.

Dynamic quantum circuit has great advantage in reducing the # qubits

Symmetry has great advantage in reducing the complexity of circuit compilation

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Thanks for your attention!

2310.11021 & PRApplied 22, 024029

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